The Pandexit model: states, equations, and mobility choice

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This is a companion piece to COVID-19 Pandexit and the effects on economic activity, covering the underlying epidemiological model in full.

This is a restatement of the core framework in Rungcharoenkitkul (2021), sections 2.1–2.2, checked against the author's public implementation. The notation below groups the six isolation equations to make the population accounting easier to follow.

States and notation

Each state is a number of people; one period is one day. Write $\Delta X_{t+1} = X_{t+1} - X_t$.

State	Interpretation
S_t	Susceptible
E_t	Exposed, not yet infectious
I_t	Infectious and able to transmit
$U_t^R,\ Q_t^R,\ H_t^R$	Isolated and eventually recovering: undetected, quarantined, hospitalised
$U_t^D,\ Q_t^D,\ H_t^D$	Isolated and eventually dying, in the same three settings
R_t	Recovered and immune
D_t	Cumulative deaths
V_t	Effectively vaccinated and immune

The twelve compartments sum to the initial population N, including deaths. The public code also tracks auxiliary series, such as cumulative detected cases; these are not additional mutually exclusive population compartments.

Infection and vaccination

Let F_t be new exposures and ν_t the daily flow of susceptible people gaining vaccine protection. With σ the incubation-transition rate, δ the isolation-transition rate, and ω the immunity-loss rate:

$$F_t = \gamma_t \frac{S_t I_t}{N}, \qquad \begin{aligned} \Delta S_{t+1} &= -F_t - \nu_t + \omega(R_t + V_t), \\ \Delta E_{t+1} &= F_t - \sigma E_t, \\ \Delta I_{t+1} &= \sigma E_t - \delta I_t, \\ \Delta V_{t+1} &= \nu_t - \omega V_t. \end{aligned}$$

Here ν_t measures effective protection, not injections: an efficacy assumption scales the vaccination input. A delay can be represented in that input without adding compartments. The displayed core equations allocate

vaccination to susceptible people; the upstream code also offers allocation across susceptible and recovered people. Setting $\omega = 0$ rules out waning immunity. Flows must be bounded so no compartment becomes negative.

Isolation, recovery, and deaths

Let q be detection probability, h hospitalisation probability conditional on detection, and p_t eventual death probability after infection. Define the allocation weights

$$w_U = 1 - q, \qquad w_Q = q(1-h), \qquad w_H = qh.$$

For each $j \in \{U, Q, H\}$, the two equations

$$\begin{aligned} \Delta j^R_{t+1} &= \delta(1-p_t) w_j I_t - \rho_j j^R_t, \\ \Delta j^D_{t+1} &= \delta p_t w_j I_t - \mu j^D_t \end{aligned}$$

represent six state transitions. The recovery rates satisfy $\rho_U = \rho_Q = \rho$, while hospital recovery uses ρ_H; μ is the death-transition rate. The remaining two equations are

$$\begin{aligned} \Delta R_{t+1} &= \rho(U^R_t + Q^R_t) + \rho_H H^R_t - \omega R_t, \\ \Delta D_{t+1} &= \mu(U^D_t + Q^D_t + H^D_t). \end{aligned}$$

Summing all twelve changes gives zero. The outcome superscripts are accounting categories, not assumptions that an individual's outcome is observed in advance.

Mobility and the policy rule

Mobility $m_t \in [-1, 0]$ is a fractional deviation from normal: -0.1 means 10% below normal. Its contribution to transmission is exponential:

$$\gamma_t(m_t) = \beta_0 e^{\beta_1 m_t} + z_t, \qquad \beta_0, \beta_1 > 0.$$

The residual z_t captures transmission changes not explained by mobility. The paper's baseline projection uses a mean-reverting residual, $z_t = \varrho z_{t-1} + \varepsilon_t$, with future innovations set to zero. This still allows the inherited residual to decay gradually.

Writing $s_t = S_t/N$ and $i_t = I_t/N$, the reduced policy block uses the mortality-loss measure

$$\tilde{d}_t(m) = p_t \delta i_t \left[1 - \delta + \gamma_t(m) s_t\right].$$

This is the policy rule's mortality measure, **not** observed daily deaths $\Delta D_{t+1}/N$. The mobility decision balances it against the cost of restrictions:

$$m^*_t \in \operatorname*{argmin}_{-1 \le m \le 0} \left\{ \tilde{d}_t(m)^2 + \varphi m^2 \right\}, \qquad \varphi > 0.$$

An interior choice satisfies

$$-\varphi m_t = \tilde{d}_t(m_t) \underbrace{p_t \delta i_t s_t \beta_0 \beta_1 e^{\beta_1 m_t}}_{\partial \tilde{d}_t / \partial m_t}.$$

Boundary choices must also be considered. The upstream numerical routine searches a mobility grid for the smallest first-order-condition residual. This is the source's reduced policy rule, rather than a newly solved full dynamic-planning problem.

A useful summary of epidemic pressure is

$$\mathcal{R}_t^{\text{eff}} = \frac{\gamma_t}{\delta}\frac{S_t}{N}.$$

Reducing mobility lowers γ_t; effective vaccination lowers S_t. Both can push this quantity below one, when the combined exposed-and-infectious pool starts shrinking.

From mobility to activity

The activity calculation is a separate empirical step. The rolling regressions below estimate an intercept and a mobility coefficient using eight monthly observations. For a scenario comparison, the approximation is $\Delta y_t \simeq \widehat{b}_t \Delta m_t$, with mobility and activity changes expressed in consistent percentage-point units. It is an estimated association, not an additional epidemiological equation or a causal identification result.

The 95%, 60%, and 5% mobility paths keep the vaccination scenario comparable while varying assumed protection against infection. This section documents their model framework; it does not supply a new calibration or re-estimate the historical charts.

References

Rungcharoenkitkul, P. (2021): Macroeconomic Consequences of Pandexit, BIS Working Papers, Bank for International Settlements.