

The Pandexit model: states, equations, and mobility choice

Nicolas Franz-Pattillo

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This is a companion piece to [COVID-19 Pandexit and the effects on economic activity](#), covering the underlying epidemiological model in full.

This is a restatement of the core framework in Rungcharoenkitkul (2021), sections 2.1–2.2, checked against the author’s [public implementation](#). The notation below groups the six isolation equations to make the population accounting easier to follow.

States and notation

Each state is a number of people; one period is one day. Write $\Delta X_{t+1} = X_{t+1} - X_t$.

State	Interpretation
S_t	Susceptible
E_t	Exposed, not yet infectious
I_t	Infectious and able to transmit
U_t^R, Q_t^R, H_t^R	Isolated and eventually recovering: undetected, quarantined, hospitalised
U_t^D, Q_t^D, H_t^D	Isolated and eventually dying, in the same three settings
R_t	Recovered and immune
D_t	Cumulative deaths
V_t	Effectively vaccinated and immune

The twelve compartments sum to the initial population N , including deaths. The public code also tracks auxiliary series, such as cumulative detected cases; these are not additional mutually exclusive population compartments.

Infection and vaccination

Let F_t be new exposures and ν_t the daily flow of susceptible people gaining vaccine protection. With σ the incubation-transition rate, δ the isolation-transition rate, and ω the immunity-loss rate:

$$\begin{aligned} \Delta S_{t+1} &= -F_t - \nu_t + \omega(R_t + V_t), \\ F_t &= \gamma_t \frac{S_t I_t}{N}, & \Delta E_{t+1} &= F_t - \sigma E_t, \\ & & \Delta I_{t+1} &= \sigma E_t - \delta I_t, \\ & & \Delta V_{t+1} &= \nu_t - \omega V_t. \end{aligned}$$

Here ν_t measures effective protection, not injections: an efficacy assumption scales the vaccination input. A delay can be represented in that input without adding compartments. The displayed core equations allocate

vaccination to susceptible people; the upstream code also offers allocation across susceptible and recovered people. Setting $\omega = 0$ rules out waning immunity. Flows must be bounded so no compartment becomes negative.

Isolation, recovery, and deaths

Let q be detection probability, h hospitalisation probability conditional on detection, and p_t eventual death probability after infection. Define the allocation weights

$$w_U = 1 - q, \quad w_Q = q(1 - h), \quad w_H = qh.$$

For each $j \in \{U, Q, H\}$, the two equations

$$\begin{aligned} \Delta j_{t+1}^R &= \delta(1 - p_t)w_j I_t - \rho_j j_t^R, \\ \Delta j_{t+1}^D &= \delta p_t w_j I_t - \mu j_t^D \end{aligned}$$

represent six state transitions. The recovery rates satisfy $\rho_U = \rho_Q = \rho$, while hospital recovery uses ρ_H ; μ is the death-transition rate. The remaining two equations are

$$\begin{aligned} \Delta R_{t+1} &= \rho(U_t^R + Q_t^R) + \rho_H H_t^R - \omega R_t, \\ \Delta D_{t+1} &= \mu(U_t^D + Q_t^D + H_t^D). \end{aligned}$$

Summing all twelve changes gives zero. The outcome superscripts are accounting categories, not assumptions that an individual's outcome is observed in advance.

Mobility and the policy rule

Mobility $m_t \in [-1, 0]$ is a fractional deviation from normal: -0.1 means 10% below normal. Its contribution to transmission is exponential:

$$\gamma_t(m_t) = \beta_0 e^{\beta_1 m_t} + z_t, \quad \beta_0, \beta_1 > 0.$$

The residual z_t captures transmission changes not explained by mobility. The paper's baseline projection uses a mean-reverting residual, $z_t = \varrho z_{t-1} + \varepsilon_t$, with future innovations set to zero. This still allows the inherited residual to decay gradually.

Writing $s_t = S_t/N$ and $i_t = I_t/N$, the reduced policy block uses the mortality-loss measure

$$\tilde{d}_t(m) = p_t \delta i_t [1 - \delta + \gamma_t(m) s_t].$$

This is the policy rule's mortality measure, **not** observed daily deaths $\Delta D_{t+1}/N$. The mobility decision balances it against the cost of restrictions:

$$m_t^* \in \operatorname{argmin}_{-1 \leq m \leq 0} \left\{ \tilde{d}_t(m)^2 + \varphi m^2 \right\}, \quad \varphi > 0.$$

An interior choice satisfies

$$-\varphi m_t = \tilde{d}_t(m_t) \underbrace{p_t \delta i_t s_t \beta_0 \beta_1 e^{\beta_1 m_t}}_{\partial \tilde{d}_t / \partial m_t}.$$

Boundary choices must also be considered. The upstream numerical routine searches a mobility grid for the smallest first-order-condition residual. This is the source's reduced policy rule, rather than a newly solved full dynamic-planning problem.

A useful summary of epidemic pressure is

$$\mathcal{R}_t^{\text{eff}} = \frac{\gamma_t}{\delta} \frac{S_t}{N}.$$

Reducing mobility lowers γ_t ; effective vaccination lowers S_t . Both can push this quantity below one, when the combined exposed-and-infectious pool starts shrinking.

From mobility to activity

The activity calculation is a separate empirical step. The rolling regressions below estimate an intercept and a mobility coefficient using eight monthly observations. For a scenario comparison, the approximation is $\Delta y_t \simeq \hat{b}_t \Delta m_t$, with mobility and activity changes expressed in consistent percentage-point units. It is an estimated association, not an additional epidemiological equation or a causal identification result.

The 95%, 60%, and 5% mobility paths keep the vaccination scenario comparable while varying assumed protection against infection. This section documents their model framework; it does not supply a new calibration or re-estimate the historical charts.

References

RUNGCHAROENKITKUL, P. (2021): [Macroeconomic Consequences of Pandexit](#), BIS Working Papers, Bank for International Settlements.