# Consumption and Savings

ECON 302, Intermediate Macroeconomics


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# This set of slides

# Introduction

# Introduction

- Consumption is a large component of GDP expenditure.
- Classic Keynesian used a linear consumption function $c_t = c_0 + c_1 Y_t^d$ to explain short term fluctuations.
- For $c_0 > 0$ and $0 < c_1 < 1$, the share of income consumed should decline as income increases (short vs long-run and the Kuznets paradox).
- In the Solow-Swan model (long-run growth) the saving rate is constant.
- We need to build a model that captures what we see in the data.

# Introduction: goals

- At the end of this lecture you should be able to understand the basics of the household dynamic problem : sequential and Life-time budget constraints and utility maximization

- You should be able to understand what is the link between consumption and savings.

- It should be clear what are the forces behind consumption smoothing.

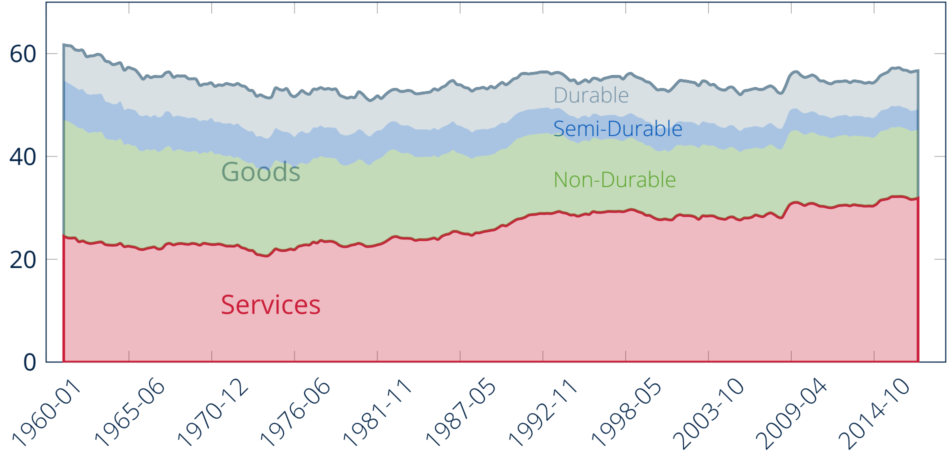
Most of the contents of this course can be found in

- Abel et al. (2018, Ch 4)
- Mankiw (2009, ch 3.3 and 16)

There's more relevant facts that involve micro-data and more sophisticated models to account for those facts. If you want to go deeper on this I suggest you to check

- Attanasio (1999)
- Romer (2018, ch 8)

**Figure 1:** Canadian Consumption (% of GDP)



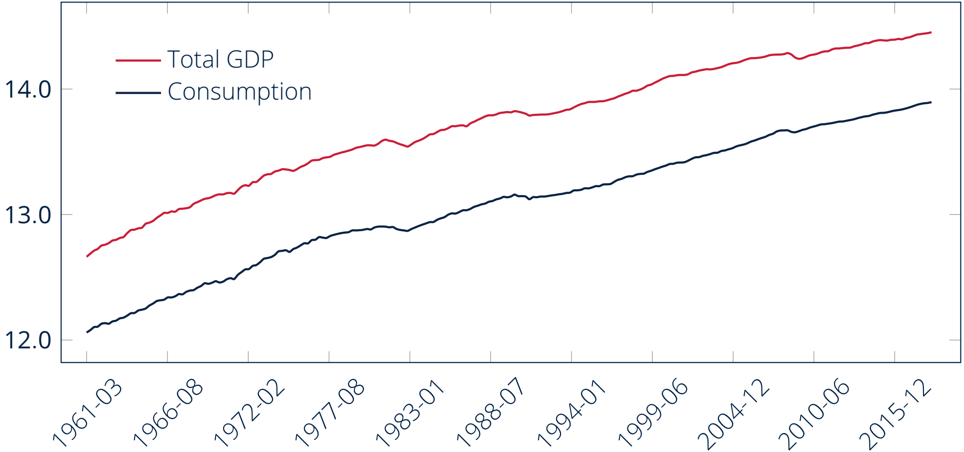

*Source*: Statistics Canada

## Durable goods

"A durable good is one that may be used repeatedly or continuously for production or consumption over a period of more than a year, assuming a normal or average rate of physical usage. The distinction between durable and non-durable goods is not based on physical durability as such. Instead, the distinction is based on whether the goods can be used once only for purposes of production or consumption or whether they can be used repeatedly, or continuously. For example, coal is a highly durable good in a physical sense, but it can be burnt only once."

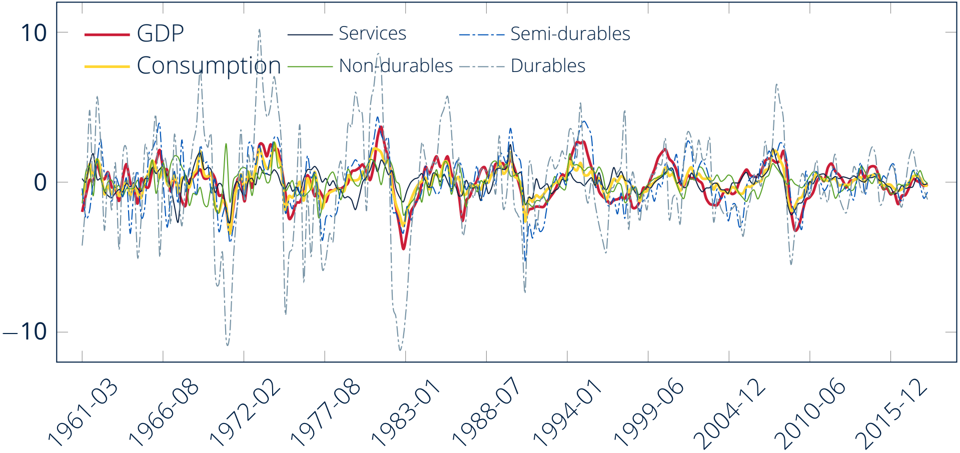
— System of National Accounts 2008, paragraph 9.42

**Figure 2:** GDP and consumption expenditures (logs)




*Source*: Statistics Canada and own calculations

**Figure 3:** Cyclical component of selected aggregates (%)

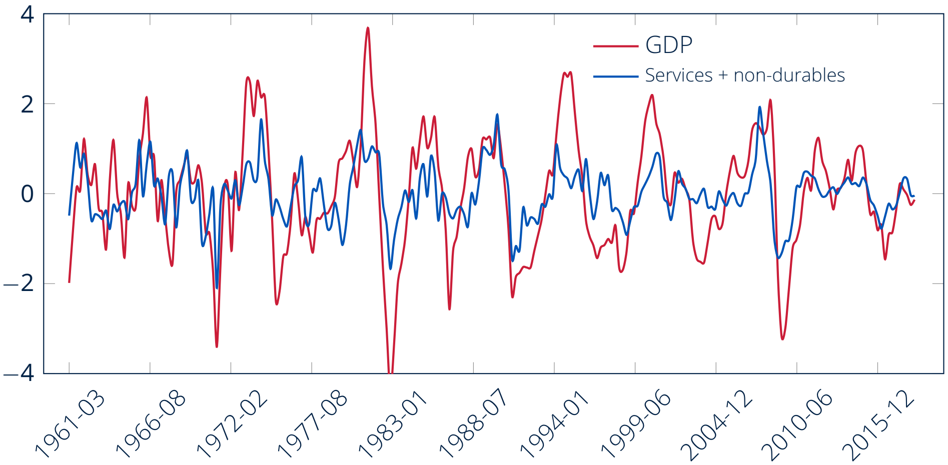



*Source*: Statistics Canada and own calculations.

- The decision of buying durable goods is more complicated than non-durables:
    - Once we get a fridge, we can enjoy its services today and tomorrow
    - The decision of renewing it might be discrete (we cannot repair it to make it look as new)
- We will focus in non-durables consumption — for a simple model with durables check Mankiw (1982).

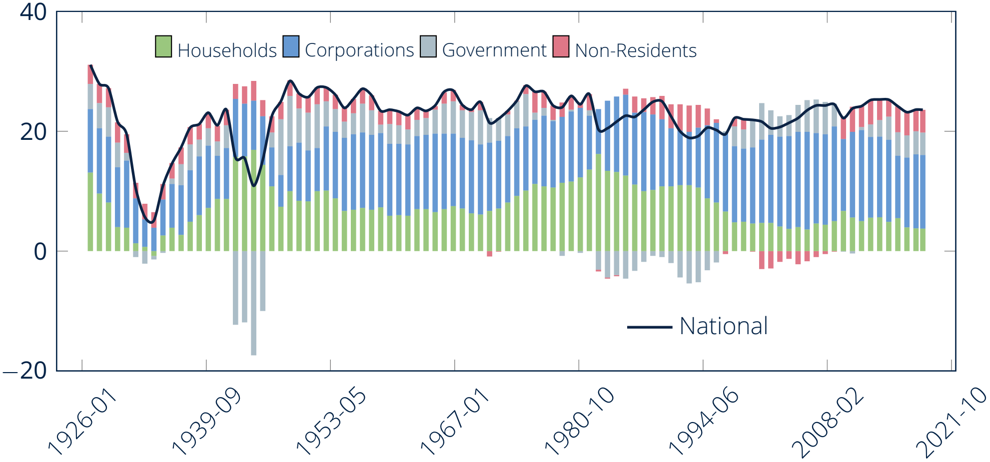
# Introduction: Facts

**Figure 4:** Cyclical component of selected aggregates (%)



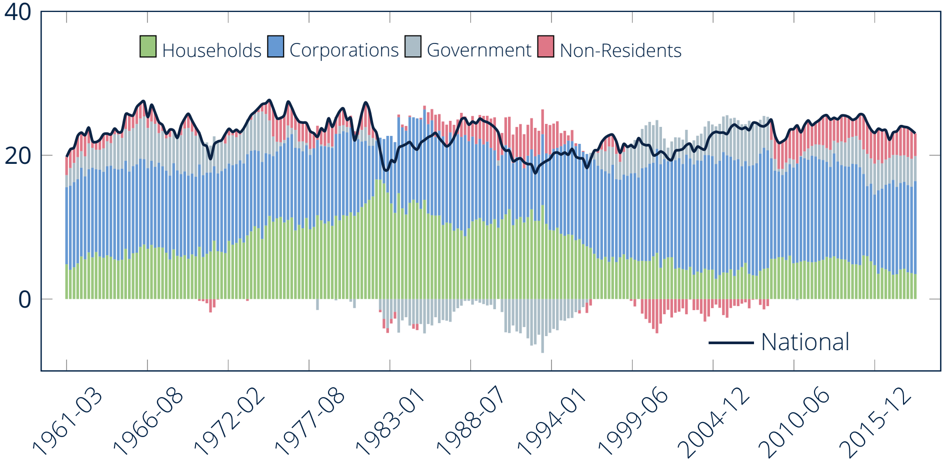

*Source*: Statistics Canada and own calculations

**Figure 5:** Canadian gross saving rate (% of GNI)




*Source*: Statistics Canada, annual data.

**Figure 6:** Canadian gross saving rate (% of GNI)




*Source*: Statistics Canada, quarterly data.

# Short math review

# Short math review: infinite sums

- Let $s = \sum_{t=0}^{\infty} \beta^t$ with $\beta \in (0,1)$. Then:

$$\begin{aligned} s &= 1 + \beta + \beta^2 + \cdots \\ \beta \cdot s &= \beta + \beta^2 + \beta^3 + \cdots \end{aligned}$$

- Since $\beta^t$ gets smaller with bigger $t$ we can subtract both to get

$$s(1-\beta) = 1$$

- Therefore

$$\sum_{t=0}^{\infty} \beta^t = \frac{1}{1-\beta}$$

# Short math review: constrained optimization

- Assume you have a multivariate function $F(x_1, \cdots, x_N)$ you would like to maximize.
- Let $\sum_{n=1}^{N} a_n x_n = A$ and $\sum_{n=1}^{N} b_n x_n = B$ two constraints on $x_n$
- The problem you would like to solve is

$$\max_{\{x_n\}_{n=1}^{N}} F(x_1, \cdots, x_N)$$

subject to

$$\sum_{n=1}^{N} a_n x_n = A \quad \text{and} \quad \sum_{n=1}^{N} b_n x_n = B$$

## Short math review: constrained optimization

- The Lagrangean of the previous problem is

$$\begin{aligned}\mathcal{L}(x_1,\cdots,x_N,\lambda_1,\lambda_2) =& F(x_1,\cdots,x_N) \\ & -\lambda_1\left[\sum_{n=1}^{N} a_n x_n - A\right] - \lambda_2\left[\sum_{n=1}^{N} b_n x_n - B\right]\end{aligned}$$

- The first order conditions are:

$$\begin{aligned}\frac{\partial\mathcal{L}(x_1,\cdots,x_N,\lambda_1,\lambda_2)}{\partial x_n} &= 0 \qquad \forall\, n\in\{1,\cdots,N\} \\ \frac{\partial\mathcal{L}(x_1,\cdots,x_N,\lambda_1,\lambda_2)}{\partial \lambda_i} &= 0 \qquad \forall\, i\in\{1,2\}\end{aligned}$$

- The solution is found by solving the system of equations ($N+2$ equations and unknowns) given by the first order conditions

# Model

How do we decide how much to consume?

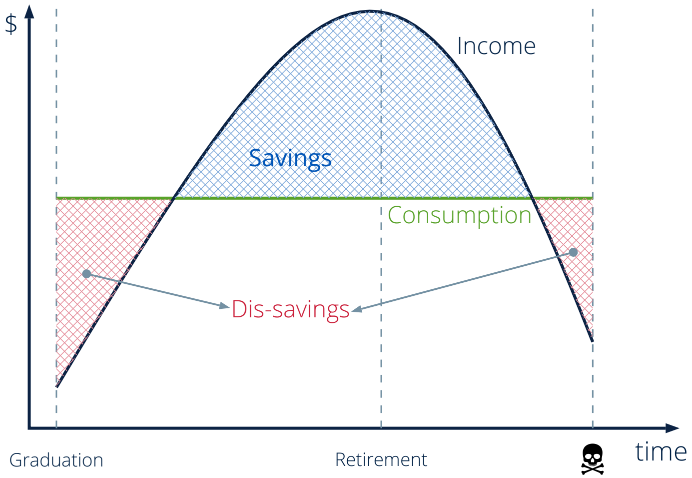
- Friedman (1957) Permanent Income Hypothesis (PIH)
- Modigliani and Brumberg (1954) Life Cycle Hypothesis (LCH)[1]

The main difference between both is the time horizon of the consumer.

[1] For a summary of Modigliani's contribution see Deaton (2005)

# Modigliani's life cycle hypothesis

**Figure 7:** Life Cycle Diagram



Why do we borrow/save?

1. Consumption smoothing. We don't like to consume a lot today and nothing tomorrow or vice-versa
2. Patience. The way we value today vs tomorrow
3. Precaution. If future income is uncertain, we might want to save to secure a minimum level of consumption
4. Life cycle. Retirement, bequest.

We will assume there's one *infinitely-lived representative household* (is this important?)

- Its objective is to maximize its lifetime utility.
- It is constrained by a budget.

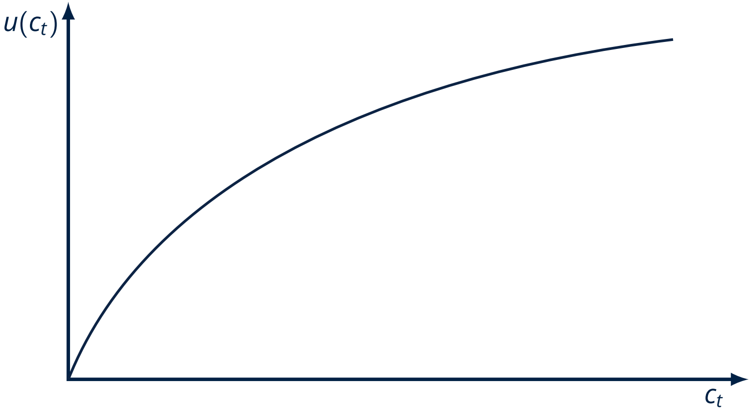
# Model: Preferences

We make a couple of assumptions regarding consumer preferences with respect to consumption:

1. More consumption is better. $u'(c) > 0$
2. The last unit is less satisfying than the one before. $u''(c) < 0$
3. When consumption approaches zero/infinite, the marginal utility approaches infinite/zero. $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$
4. Inter-temporal utility is additively separable. $\dfrac{\partial u(c_t)}{\partial c_{t+j}} = 0 \quad \forall j \neq 0$
5. A given level of consumption is more enjoyable today relative to tomorrow. A discount factor $0 < \beta \leq 1$

The first three assumptions define the shape of the *intra-temporal utility*

**Figure 8:** Utility function



# Model: Preferences

The last two assumptions allow us to write the life-time utility of the household as

$$U_0(c_0, c_1, c_2, \cdots) = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

## A —sometimes— useful simplification

Another way of looking at this aggregation is by dividing it in *present-future* instead of infinite periods. Note that (1) can be written as $U_0 = u(c_0) + \beta U_1(c_1, c_2, c_3, \cdots)$. Since the problem from $t$ to $t+1$ is the same than from $t+1$ to $t+2$, we can trivialize the futures as

$$U_0 = u(c_0) + \beta u(c_f)$$

where $c_f$ represents consumption in the future.

- What would be the optimal path of consumption if we maximize (1)?
- Since more is better, we would choose infinite consumption for all periods.
- The budget constraint states that expenditures have to be smaller or equal to revenues.
- The budget constraint consolidates the idea that when resources are limited we cannot have it all (i.e. we have to choose).

1. Assume that the household has an *exogenous* stream of income $y_t$ $\forall t$
2. The only way to carry resources to/from the futures is by saving $s_{t+1}$ and the household is born with $s_0$ in its pocket. Borrowing implies negative $s_{t+1}$. Note that $s_t$ is a state variable while $s_{t+1}$ is the level of savings chosen in period $t$ to be carried to $t+1$.
3. The interest rate is constant ($r$) and the price of the consumption good is $p_t = 1$ (Is this important?)

**Sequential budget constraint**

With the assumptions above, each period the household balances expenditures with its sources of income:

$$c_t + s_{t+1} = y_t + (1+r)s_t \quad \forall\, t \tag{2}$$

Different environments lead to different budget constraints!

# Model: budget constraint

Rearranging (2) and remembering that it holds for all periods we get

$$\begin{aligned}
s_0 &= \frac{s_1 + c_0 - y_0}{(1+r)} \\
s_1 &= \frac{s_2 + c_1 - y_1}{(1+r)} \\
&\vdots \\
s_t &= \frac{s_{t+1} + c_t - y_t}{(1+r)} \\
s_{t+1} &= \frac{s_{t+2} + c_{t+1} - y_{t+1}}{(1+r)} \\
s_{t+2} &= \frac{s_{t+3} + c_{t+2} - y_{t+2}}{(1+r)} \\
&\vdots
\end{aligned}$$

# Model: budget constraint

Substituting the values of $s_{t+j}$ in $s_t$ and stopping at an arbitrary $T$ we get

$$s_0 = \frac{s_{T+1}}{(1+r)^t} + \sum_{t=0}^{T} \frac{c_t - y_t}{(1+r)^t}$$

## No-Ponzi scheme condition

In a Ponzi scheme the debt grows infinitely. This cannot happen since the resources devoted to it will run out. Therefore, household's debt cannot explode. On the other side, there's no benefit of saving infinitely and therefore household's saving cannot explode either (what would happen with the interest rate?). Since $s_t$ is bounded we get to the conclusion:

$$\lim_{T\to\infty} \frac{s_{T+1}}{(1+r)^T} = 0$$

# Model: budget constraint

## Life-time budget constraint

Once the Ponzi-schemes are ruled out, the combination of the sequential budget constraints yield the life-time budget constraint:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = s_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \tag{3}$$

which implies that the present value of all expenditure in consumption has to equalize the present value of all sources of income (initial endowment and exogenous income)

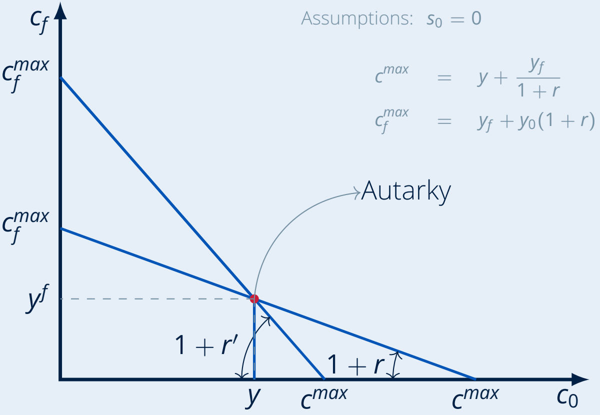
## Present value of life-time resources

$$PVLR = s_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \tag{4}$$

# Example

## Two period budget visual representation

**Figure 9:** Budget constraint



# Model: consumer's problem

Given the preferences, budget constraint, initial condition, and no-Ponzi scheme condition, the consumer's problem can be written as:

$$\max_{\{c_t, s_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to:

$$c_t + s_{t+1} = y_t + (1+r)s_t \quad \forall\, t$$

$$s_0 \text{ given}$$

$$\lim_{t\to\infty} \frac{s_{t+1}}{(1+r)^t} = 0$$

# Model: consumer's problem (second version)

Given the preferences, budget constraint, initial condition, and no-Ponzi scheme condition, the consumer's problem can also be written as:

$$\max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = PVLR$$

How does the solution look like?

- If there's no uncertainty the solution is a path for consumption and savings from $t = 0$ to infinity.
- If the problem involves uncertainty, then the solution is a contingent plan that depends on the realization of the world (i.e. a consumption level for all scenarios and time).

**Winning the lottery?**

Let's assume that there's a considerable chance that you might win the lottery. Would you adjust your current consumption if the probability of winning increases significantly? How would you adjust your consumption later on if you actually win it versus if you don't? would it be the same in both scenarios?

# Model: consumer's problem

Ignoring uncertainty[2] the Lagrangean of the problem's first version is

$$\mathcal{L}_1\left(c_t, s_{t+1}, \lambda_t\right) = \sum_{t=0}^{\infty} \beta^t \left\{u(c_t) - \lambda_t \left[c_t + s_{t+1} - y_t - (1+r)s_t\right]\right\} \tag{5}$$

The Lagrangean of the problem's second version is

$$\mathcal{L}_2\left(c_t, \lambda\right) = \left\{\sum_{t=0}^{\infty} \beta^t u(c_t) - \lambda \left[\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} - PVLR\right]\right\} \tag{6}$$

Since both problems lead to the same solution, we will show the easiest: *version 2* in (6)

[2] I will describe the solution with uncertainty in the optional material at the end.

The first order conditions of consumer's problem (version 2) are

$$
\begin{aligned}
\frac{\partial \mathcal{L}_2}{\partial c_0} = 0 &\implies \qquad u'(c_0) = \lambda \\
&\vdots \\
\frac{\partial \mathcal{L}_2}{\partial c_t} = 0 &\implies \qquad \beta^t u'(c_t) = \lambda/(1+r)^t \\
\frac{\partial \mathcal{L}_2}{\partial c_{t+1}} = 0 &\implies \beta^{t+1} u'(c_{t+1}) = \lambda/(1+r)^{t+1} \\
&\vdots \\
\frac{\partial \mathcal{L}_2}{\partial \lambda} = 0 &\implies \sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = PVLR
\end{aligned}
$$

- Note that the Lagrange multiplier is common across all equations (and strictly positive). Taking the F.O.C.'s for an arbitrary $t$ and dividing its equivalent in $t+1$ we get:

$$u'(c_t) = (1+r)\beta u'(c_{t+1}) \qquad \forall\, t \tag{Euler eq}$$

- The derivative of the Lagrangean with respect to the multiplier returns the lifetime budget constraint:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = s_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \tag{LT budget}$$

- Euler eq and LT budget solves for the path of consumption (?)

# Model: Euler Equation

Euler eq defines the optimal relation between $c_t$ and $c_{t+1}$ and, therefore, states the consumption-saving trade-off

- Assume you have some consumption plan $\tilde{c}_t$ and $\tilde{c}_{t+1}$
- Now let's say you consider saving a bit more today (decrease $\tilde{c}_t$)
- What is the cost? the marginal utility from consumption at $\tilde{c}_t$
- What is the benefit? save implies receiving $(1+r)$ for every unit saved $\implies$ consumption will increase utility by the (discounted) marginal utility at $\tilde{c}_{t+1}$
- If such reallocation of consumption improves your lifetime utility (i.e LHS$<$RHS), then it means that the current plan is not optimal.

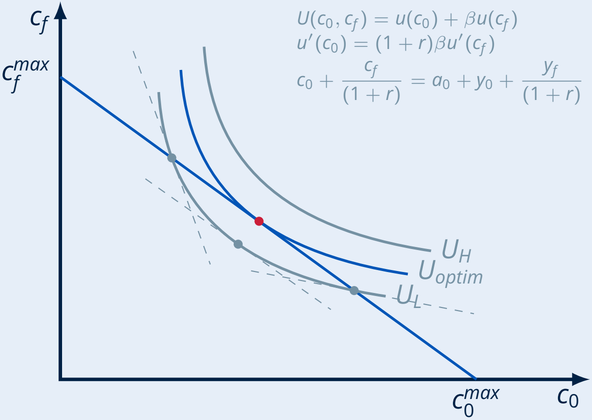
## Two-periods

Assume this economy lasts for two periods. Denote consumption in the present as $c_0$ and in the future by $c_f$. Similarly, present and future income are denoted by $y_0$ and $y_f$, respectively. Given $s_0$ and the no-Ponzi scheme condition (?), the solution for the consumer problem is defined by (two equations and two unknowns):

$$\begin{aligned} u'(c_0) &= (1+r)\beta u'(c_f) \\ c_0 + \frac{c_f}{(1+r)} &= s_0 + y_0 + \frac{y_f}{(1+r)} \end{aligned}$$

## Two-periods: graphical representation

**Figure 10:** Two periods solution



## Infinite periods and log utility

Assume the consumer lives for infinite periods and the intra-temporal utility function is natural logarithm. Under these conditions, the Euler eq implies

$$c_{t+1} = (1+r)\beta c_t$$

Then $c_t = c_0(1+r)^t\beta^t$. Using this in the budget constraint solves for $c_0$

$$\begin{aligned} \sum_{t=0}^{\infty} \frac{c_0(1+r)^t\beta^t}{(1+r)^t} &= PVLR \\ \implies \qquad c_0 &= \frac{PVLR}{\sum_{t=0}^{\infty}\beta^t} = (1-\beta)PVLR \end{aligned}$$

With $c_0$ in hand we recover the value of $c_t$ for any $t$.

# Consumption dynamics

# Market prices and patience

With information about $\beta$ (level of patience) and $r$ (market price of loanable funds), and assuming $u''(c) < 0$, we have:

- Market price exactly compensates impatience: $\beta(1+r) = 1$

$$u'(c_t) = u'(c_{t+1}) \Leftrightarrow c_t = c_{t+1}$$

- Market price “over-compensates” impatience: $\beta(1+r) > 1$

$$u'(c_t) > u'(c_{t+1}) \Leftrightarrow c_t < c_{t+1}$$

- Market price “under-compensates” impatience: $\beta(1+r) < 1$

$$u'(c_t) < u'(c_{t+1}) \Leftrightarrow c_t > c_{t+1}$$

# Example

## Market price exactly compensates for impatience

- Assume $\beta(1+r) = 1$
- Choose any strictly concave utility
- Assume some values for $y_t \; \forall \; t$ and $s_0$
- Euler equation implies $\bar{c} = c_t = c_{t+1}$
- Substitute into the budget constraint:

$$
\begin{aligned}
\sum_{t=0}^{\infty} \frac{\bar{c}}{(1+r)^t} &= PVLR \\
\bar{c} \cdot \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} &= PVLR \\
\bar{c} &= \frac{r}{1+r} PVLR
\end{aligned}
$$

# Changes in income profile

- Suppose $y_{t+k}$ increases
- total expenditure on consumption must increase. (why?)
- if we start with some optimal plan, and just change $c_{t+k}$, but not $c_{t+k+1}$, this will violate the Euler eq.
- This is **consumption smoothing**.

**Consumption smoothing**

The desire to obtain a more stable and predictable consumption path compared to income. It implies that an increase of current consumption must be accompanied by an increase of future consumption (not necessarily by the same magnitude).

- Suppose $y_{t+k}$ increases
- total expenditure on consumption must increase. (why?)
- The way the path of consumption will look depends on:
  - Is the change expected?
  - Is the change in income permanent or transitory

# Example

## Permanent vs. transitory and anticipated vs. unanticipated income increase

- For simplicity, let market price exactly compensate for impatience and no initial endowment: $\beta(1+r) = 1$ and $s_0 = 0$
- What is the path of consumption if:
  1. Income is constant and equal to $\bar{y}$?
  2. It is anticipated, at time zero, that income will increase by $\epsilon$ next period, temporarily. $y_1 = \bar{y} + \epsilon$ and $y_t = \bar{y} \; \forall \; t \neq 1$
  3. The same increase is now permanent. $y_0 = \bar{y}$ and $y_t = \bar{y} + \epsilon \; \forall \; t \geq 1$
  4. The income increase is unanticipated and transitory.
  5. The unanticipated change is permanent .

## Permanent vs. transitory and anticipated vs. unanticipated income increase (cont.)

1. Constant $\bar{y}$ will imply:

$$c_t = \bar{y} \qquad \forall\, t$$

2. Anticipated and temporary:

$$c_t = \bar{y} + \frac{r \cdot \epsilon}{(1+r)^2} \qquad \forall\, t$$

3. Anticipated and permanent:

$$c_t = \bar{y} + \frac{\epsilon}{1+r} \qquad \forall\, t$$

# Example (cont.)

## Permanent vs. transitory and anticipated vs. unanticipated income increase (cont.)

1. Unanticipated and transitory:

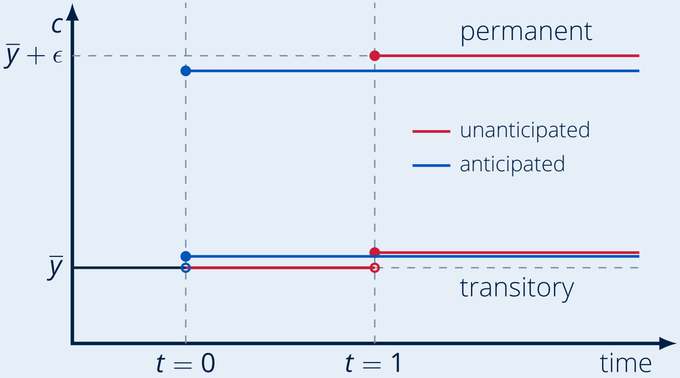
$$c_t = \begin{cases} \bar{y} & \text{if } t = 0 \\ \bar{y} + \dfrac{r \cdot \epsilon}{1+r} & \text{if } t \geq 1 \end{cases}$$

2. Unanticipated and permanent :

$$c_t = \begin{cases} \bar{y} & \text{if } t = 0 \\ \bar{y} + \epsilon & \text{if } t \geq 1 \end{cases}$$

# Example (cont.)

## Permanent vs. transitory and anticipated vs. unanticipated income increase (cont.)



# Comparative Statics for $\beta$

- Intuitively, if $\beta$ increases, we become more patient, so expect to defer more consumption to the future.

- $\beta$ is not part of the budget constraint, but is a part of the Euler equation.

$$u'(c_t) = (1 + r)\beta u'(c_{t+1})$$

- If we start with Euler equation that holds with equality, and consider higher $\beta$:
    - RHS $>$ LHS
    - Re-optimize: lower $c_t$ and higher $c_{t+1}$; i.e. increase in $u'(c_t)$ and decrease $u'(c_{t+1})$

## Comparative Statics for $r$

In general, $r$ has ambiguous effects on the consumption plan. The current position on savings matter.

- **Substitution Effect**: increases in $r$ makes current consumption less attractive.
  - Assume PVLR is not altered. Euler equation: $u'(c_t) = \beta(1+r)u'(c_{t+1})$.
  - Thus, higher $r$ implies lower $c_t$ and higher $c_{t+1}$ to re-establish equilibrium. Similarly to an increase in $\beta$
- **Income Effect**: $r$ is also part of PVLR. An increase in $r$ makes you richer or poorer depending on if you save or borrow.
  - Borrowers suffer a negative income effect: lower consumption.
  - Lenders enjoy a positive income effect: higher consumption.

## Borrowers and changes in $r$

- Let $u(c_t) = \dfrac{c^{1-\sigma} - 1}{1-\sigma} \implies u'(c) = c^{-\sigma}$
- By the Euler equation we have $\dfrac{c_{t+1}}{c_t} = [\beta(1+r)]^{1/\sigma}$
- Assume that $s_0 = 0, y_0 = 0, y_t = \bar{y} > 0$ for all $t > 0$ to ensure this consumer is a borrower.
- The optimal consumption plan is:

$$c_0 = \left\{1 - \beta^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \frac{\bar{y}}{r}\,; \qquad c_t = c_0\left[(1+r)\beta\right]^{t/\sigma}$$

- lower current consumption due to both income and substitution effects

# Example

## Lenders and changes in $r$

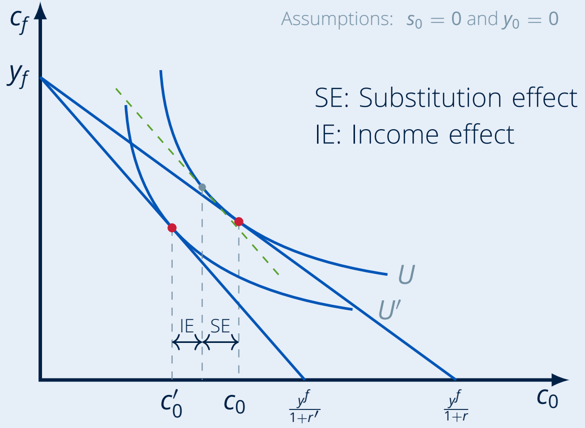
- Let $s_0 = 0$, $y_0 > 0$, $y_t = 0 \; \forall t > 0$ (Lender)
- The optimal consumption plan is:

$$c_0 = \left\{1 - \beta^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}\right\}\bar{y}\,; \qquad c_t = c_0\left[(1+r)\beta\right]^{t/\sigma}$$

- the effect of $r$ on current consumption is ambiguous:
  - decline if $\sigma < 1$ (substitution effect dominates)
  - increase if $\sigma > 1$ (income effect dominates)
  - no change if $\sigma = 1$ (IE and SE offset)

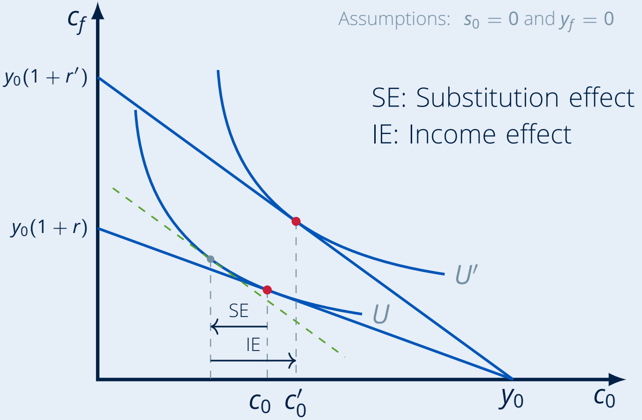
## Increase in $r$ in two periods, Borrower

**Figure 11:** Income and substitution effects on current consumption



## Increase in $r$ in two periods, Lender

**Figure 12:** Income and substitution effects on current consumption



With the path of consumption and exogenous income it is easy to get the path of savings using the sequential budget constraint.

$$s_{t+1} = s_t(1+r) + y_t - c_t$$

Therefore:

$$s_{t+1} = s_0(1+r)^{t+1} + \sum_{k=0}^{t}(1+r)^k(y_k - c_k)$$

# Government

# Government

- Why do we need to include government?
  - A direct effect on national (aggregate) saving: $S = Y - C - G$
  - Fiscal policy (taxes and government expenditures) may affect consumer decisions
- Even though the government will not be “sophisticated” in our model, we can still learn a lot

# Government: assumptions

- Each period the government has a budget constraint $G_t + (1 + r)B_t = T_t + B_{t+1}$
- What is $G$?
  - We will assume $G$ is tossed to the sea
  - More realistic features (e.g. public goods) are a bit more complicated to model.
- What is $T$? total tax revenues.
- What is $B$? government borrowing (e.g. bonds)
- The budget states that any spending that is not covered by taxes ($T$), must be covered by borrowing ($B$) (negative saving)

# Government: assumptions

- Ruling out Ponzi-schemes, the lifetime budget constraint of the government is

$$\sum_{t=0}^{\infty} \frac{G_t}{(1+r)^t} = B_0 + \sum_{t=0}^{\infty} \frac{T_t}{(1+r)^t}$$

- Tax revenues can come from:
  - lump-sum tax: based on a fixed amount. (e.g. tag fees on vehicles )
  - proportional tax: the amount of tax revenues depend on the tax rate and the tax base. (e.g. income tax or V.A.T.)

# Government: assumptions

- Household's sequential budget constraint with lump-sum taxes:

$$c_t + s_{t+1} + T_t = y_t + s_t(1+r)$$

- Household's lifetime budget constraint with lump-sum taxes:

$$\sum_{t=0}^{\infty} \frac{c_t + T_t}{(1+r)^t} = s_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}$$

- Household's sequential budget constraint with proportional tax on income:

$$c_t + s_{t+1} + \tau_t y_t = y_t + s_t(1+r)$$

- Household's lifetime budget constraint with with proportional tax on income:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = s_0 + \sum_{t=0}^{\infty} \frac{y_t(1-\tau_t)}{(1+r)^t}$$

**Ricardian equivalence**

for any given pattern of government spending ($G_t$), the method of financing it (taxes or debt) does not affect agents' consumption decisions, and thus, it does not change aggregate demand.

# Example

## Drop in taxes

- Assume current lump-sum tax $T_0 = T$ and fixed $G_t = \overline{G}$.
- The optimization of the household yields the same Euler equation (why?)
- The lifetime budget constraint of the household is

$$\sum_{t=0}^{\infty} \frac{c_t + T_t}{(1+r)^t} = s_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}$$

- Assume $T_0$ drops to $T' < T$, what will happen to consumption and aggregate output? $(Y = C + G + S)$
- Ricardian equivalence $\implies$ Nothing.

# Example (cont.)

## Drop in taxes (cont.)

- If the government will keep its consumption plan, less taxes imply Borrowing to the public.
- The government's budget constraint in the future imply that more taxes will be collected in the future.
- The household take this into account when the reduction in taxes happens and saves all the tax reduction.
- This becomes obvious by replacing the government's budget into the household's lifetime budget constraint (nothing changes to the PVLR):

$$\sum_{t=0}^{\infty} \frac{c_t + \overline{G}}{(1+r)^t} = s_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}$$

Why does the Ricardian Equivalence hold in the model?

- Consumers understand that the government is subject to a budget constraint.
- Consumer are forward looking, and so they know that a current tax reduction must imply a future tax increase
- The tax does not affect the Euler equation. Would a proportional tax on income change things?

Does the Ricardian Equivalence hold in reality?

- Inter-generational considerations (e.g we know taxes will increase in the future, but what if it happens when the next generation has to pay the bill?)
- Consumer may not be as rational as we assume they are (e.g. they may not fully understand the government budget constraint)
- borrowing/liquidity constrained consumers

# Summary

# Summary

- The consumer's dynamic problem consist in the maximization of utility given a budget.
- The solution implies a path for consumption which is reflected in the Euler eq and the LT budget.
- The connection between period $t$ and $t+1$ is the ability of save or borrow. If saving/borrowing is not possible, then the solution is trivial: autarky
- Given a consumption path, saving or borrowing is a consequence of such consumption and the income process.
- The consumer smooths consumption and incorporate all news about the future immediately.
- More patient consumers will consume less now and more in the future than the less patient.

# Summary

- Our model can reproduce all motives for saving except *precaution*. For this we would need to include uncertainty
- Changes in current interest rate affect current consumption from borrowers and lenders differently:
  - substitution effect (always negative)
  - income effect (positive or negative)
- Government policy might or might not affect private consumption depending on if the Ricardian equivalence holds or not.
  - **Key assumptions:** forward looking agents, knowledge of the government budget constraint and undisturbed Euler equation.
  - Lower taxes now with no changes in government expenditures will imply higher taxes in the future, therefore consumer saves exactly the same amount that the government is borrowing

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# Additional material

# Additional material

- In some of the slides I mention expectations and uncertainty.
- As we saw, consumption decisions depend on what happens in the future.
- Since we don't know what will happen with certainty, our decisions depend on what we "expect" from the future.
- These slides will elaborate more on this topic but I won't expect from you to master it (i.e. optional)

- Let $X$ be a random variable with $J$ number of finite possible outcomes.
- Denote each of those outcomes as $x_j$ with $j \in \{1, 2, \cdots, J\}$.
- Assume that the probability of $x_j$ occurring is $0 < p_j < 1$: $pr\{X = xj\} = p_j$.
- The sum of all $p_j$ equals one. $\sum_{j=1}^{J} p_j = 1$.

**Expected value of $X$**

The expected value of the variable $X$ is the weighted average of all its possible outcomes, where the weight is the probability of each outcome

$$\mathbb{E}(X) = \sum_{j=1}^{J} p_j \cdot x_j$$

- Keep the assumptions on the random variable $X$.
- Let $F(X)$ be a function well defined in $x_j \; \forall \, j \in \{1, 2, \cdots, J\}$

**Expected value of $F(X)$**

The expected value of the function $F(X)$ is the weighted average of all its possible outcomes, where the weight is the probability of $x_j$ happening.

$$\mathbb{E}[F(X)] = \sum_{j=1}^{J} p_j \cdot F(x_j)$$

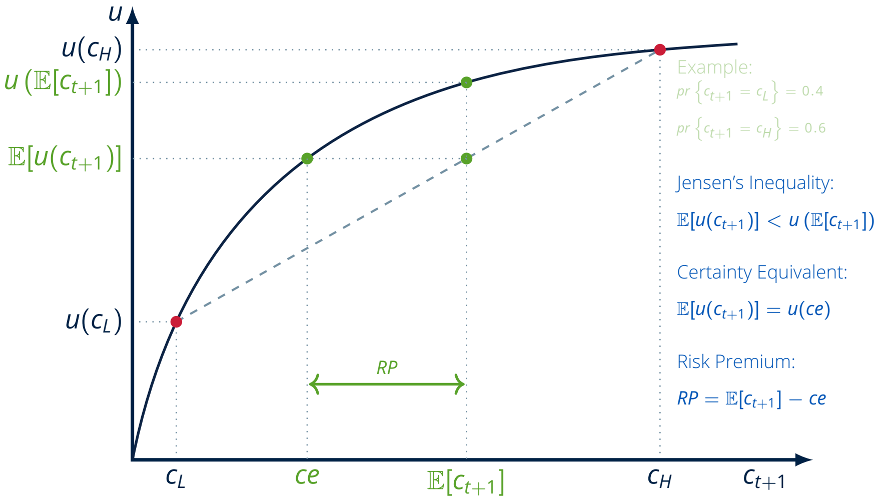
## Risk aversion

Note that the utility function is concave. This implies that any linear combination of utilities —let's say $\lambda u(c_L) + (1-\lambda)u(c_H)$ with $\lambda \in (0,1)$— will always be less preferred than the utility of the linear combination of consumptions —in our example $u(\lambda c_L + (1-\lambda)c_H)$.

This is important because if the future is uncertain, then the expected utility of two possible outcomes ($c_L$ and $c_H$) is always lower than the utility of expected consumption. Therefore the consumer would be willing to trade some of the expected consumption in order to reduce the risk. This is what we call risk aversion.

# Additional material: Preferences

**Figure 13:** Risk aversion and certainty equivalence



## Arrow-Pratt absolute and relative risk aversion measures

Absolute risk aversion

$$AR(c) = \frac{u''(c)}{u'(c)} \tag{7}$$

Relative risk aversion

$$RR(c) = c \cdot \frac{u''(c)}{u'(c)} \tag{7'}$$

## Kimball's absolute and relative prudence measures

Absolute prudence

$$AP(c) = \frac{u'''(c)}{u''(c)} \tag{8}$$

Relative prudence

$$RP(c) = c \cdot \frac{u'''(c)}{u''(c)} \tag{8'}$$

- For simplicity, assume the consumer lives from $t$ to $t+1$. From the perspective of period $t$, only the variables in $t+1$ are uncertain.
- Assume $y_{t+1}$ to be random, and could take any of the values $y_j$ with probability $p_j$, where $j \in \{1, \cdots, J\}$.
- The consumer's problem is:

$$\max_{\{c_t, c_{t+1}, s_{t+1}\}} u(c_t) + \beta \mathbb{E}_t u(c_{t+1})$$

subject to:

$$c_t + s_{t+1} = y_t + s_t(1+r)$$
$$c_{t+1} + s_{t+2} = y_{t+1} + s_{t+1}(1+r)$$

- The budget constraint in $t+1$ will force $c_{t+1}$ to take different values depending of the realization of $y_{t+1}$.
- This implies that we must consider $J$ potential budget constraints for the future.
- Taking into account the no-Ponzi scheme condition we get that with probability $p_j$ next period budget constraint will be.

$$c_j + \cancelto{0}{s_{t+2}} = y_j + s_{t+1}(1+r) \qquad \forall j$$

Substituting for the definition of expectation we get the following Lagrangean:

$$\begin{aligned}\mathcal{L} =& u(c_t) + \lambda_t[c_t + s_{t+1} - y_t + s_t(1+r)] \\ &+ \beta \sum_{j=1}^{J} p_j \left\{ u(c_j) + \lambda_j[c_j - y_j + s_{t+1}(1+r)] \right\}\end{aligned}$$

The first order condition with respect to the Lagrange multipliers will return, as usual, the sequential budget constraints.

$$\begin{aligned}c_t + s_{t+1} &= y_t + s_t(1+r) \\ c_{t+1} &= y_{t+1} + s_{t+1}(1+r)\end{aligned}$$

# Additional material: consumer's problem (uncertainty)

The first order conditions with respect $c_t$, $s_{t+1}$ and $c_j$ (i.e. any potential $c_{t+1}$) are

$$\begin{aligned} u'(c_t) &= \lambda_t \\ \lambda_t &= \beta(1+r)\sum_{j=1}^{J} p_j \lambda_j \\ u'(c_j) &= \lambda_j \end{aligned}$$

Combining the previous equations we get

$$u'(c_t) = \beta(1+r)\sum_{j=1}^{J} p_j u'(c_j)$$

where the sum in the right hand side is the definition of the expectation of $u'(c_{t+1})$

# Additional material: consumer's problem (uncertainty)

- The solution with uncertainty is described by the Euler equation:

$$u'(c_t) = (1+r)\beta\mathbb{E}_t u'(c_{t+1}) \qquad \forall\, t \tag{9}$$

- and the budget constraints:

$$\begin{aligned} c_t + s_{t+1} &= y_t + s_t(1+r) \\ c_{t+1} &= y_{t+1} + s_{t+1}(1+r) \end{aligned}$$

- The solution is current consumption ($c_t$), savings ($s_{t+1}$), and contingent future consumption ($c_{t+1} = c_j$).
- Note that this system contains $(2+J)$ equations and $(2+J)$ unknowns.

# Example

## Two periods with uncertainty and quadratic utility

- Assume the consumer has utility $u(c) = c - \frac{\gamma}{2}c^2$
- Next period income could be $y_H$ with probability $p$, or $y_L$ with probability $(1-p)$
- Note that the solution will consist in the optimal current consumption and a contingent plan for the future consumption: $(c_{f,L}, c_{f,H})$
- There is also two contingent budget constraints:

$$\begin{aligned} L: \qquad c_0 + \frac{c_{f,L}}{(1+r)} &= a_0 + y_0 + \frac{y_{f,L}}{(1+r)} \\ H: \qquad c_0 + \frac{c_{f,H}}{(1+r)} &= a_0 + y_0 + \frac{y_{f,H}}{(1+r)} \end{aligned}$$

## Two periods with uncertainty (cont.)

From the Euler equation we get

$$
\begin{aligned}
-\gamma c_0 &= \beta(1+r)\underbrace{\left[p \cdot (-\gamma c_{f,H}) + (1-p)\cdot(-\gamma c_{f,L})\right]}_{\mathbb{E}u'(c_f)} \\
c_0 &= \beta(1+r)\left[p \cdot c_{f,H} + (1-p)c_{f,L}\right]
\end{aligned}
$$

Combining the Euler equation and the budget constraint we get:

$$
c_0 = \frac{\beta(1+r)^2}{1+\beta(1+r)^2}\underbrace{\left[a_0 + y_0 + \frac{p \cdot y_{f,H} + (1-p)y_{f,L}}{1+r}\right]}_{\mathbb{E}[PVLR]}
$$

with $c_0$ we can find $c_{f,H}$ and $c_{f,H}$ using the contingent budget constraints.

# Precautionary savings

- When facing more uncertainty about the future (higher variance in the income process), some households tend to save more.
- This motive for saving is called "precautionary"
- To have precautionary savings behavior in our model we need uncertainty plus one of the following:
  1. The household is prudent ($u''' > 0$)
  2. The household has borrowing constraints