Investment

ECON 302, Intermediate Macroeconomics

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August, 2018

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This set of slides

Introduction

Introduction

- Investment corresponds to 15 to 25% of GDP for most developed economies.

- Volatility of investment is high at business cycle frequencies. Therefore, investment matters a lot for business cycle fluctuations

- Investment determines the stock of capital. It increases the productive capacity of the economy, and therefore future standard of living

Introduction: goals

- At the end of this lecture you should understand why investment is negatively related to real interest rate.
- Elements that shift the investment function.
- Dynamics of capital in a simple forward-looking environment and its relationship with investment.
- Some of the limitations of the neoclassical investment function when compared to the (micro) data.

Introduction: readings

To get a basic understanding about Investment, I suggest you to look at

- Mankiw (2009, Ch. 18)
- Abel et al. (2018, Ch. 4.2)

For a deeper and more formal analysis check:

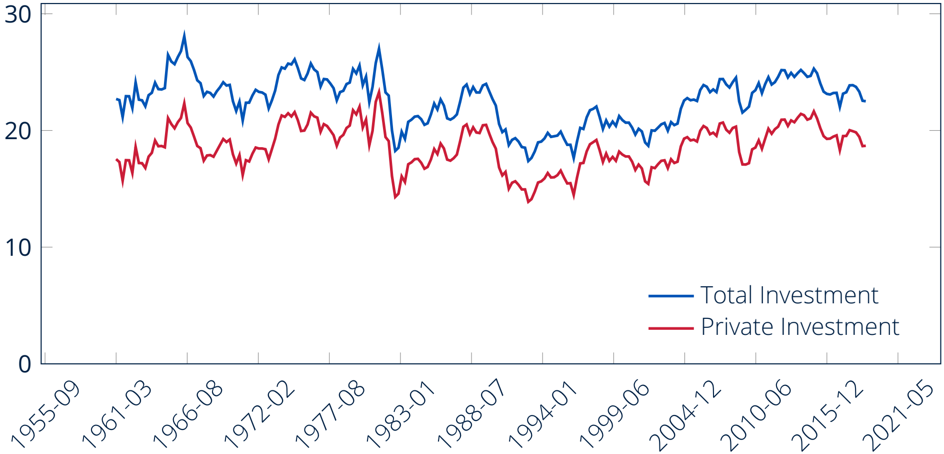
- Romer (2018, Ch. 9)
- Caballero (1999)

In our model Investment will be production or purchase of new capital goods.

In the data (Expenditure of GDP) we have three agents investing: businesses, government and non-profit institutions serving households. We will focus in businesses (why?):

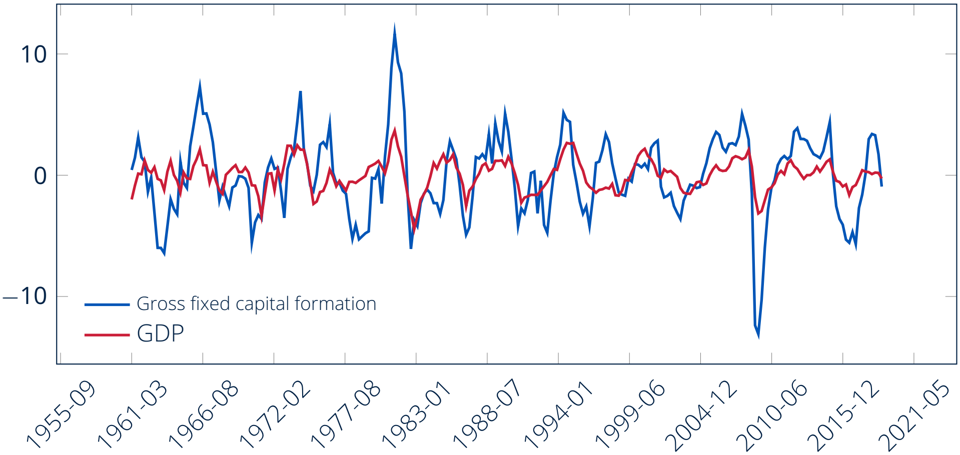
- Gross fixed capital formation
  - Residential investment
  - Non residential structures
  - Machinery and equipment
  - Intellectual property
- Change in inventories (usually small)

Figure 1: Investment (% GDP)



Source: Statistics Canada

Figure 2: GDP and GFKF cyclical components

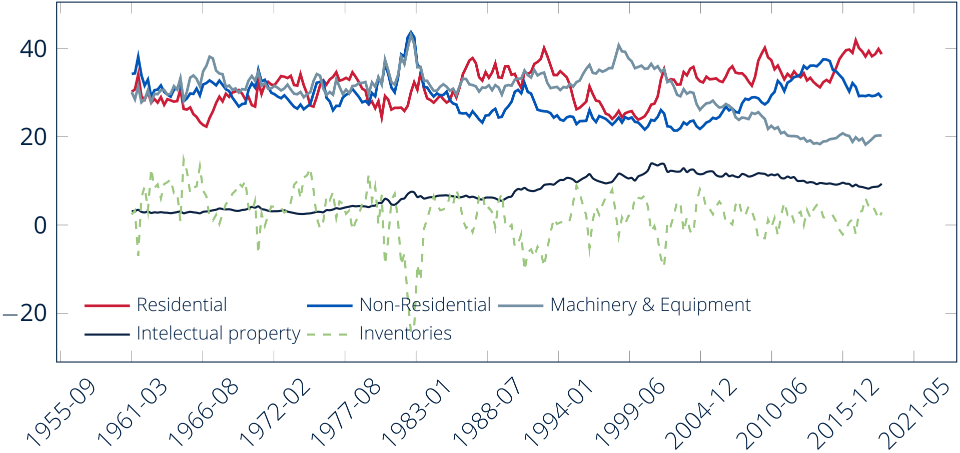


Source: Statistics Canada and own calculations

Investment is by far the most volatile component of aggregate spending:

- 3 times more volatile than GDP
- 6 times more volatile than Consumption.

Figure 3: Main components of business Investment (% of total)



Source: Statistics Canada

Model

Model: the demand for capital

Assume the following:

- Firms use the production function $A_t F(X_t, K_t)$ to convert capital K_t and other factors X_t into a final product Y_t.
- All markets are competitive.
- They rent the capital at rate r_K and other factors at rate p_X
- The price at which the output is sold is $P_t = 1$. (is this important?)

Under these assumptions the profit-maximizer firm will demand capital following

$$A_t F_K(X_t, K_t) = r_K \tag{1}$$

Marginal product of capital = rental rate of capital

Model: who supplies capital?

- We still need to know who supplies capital and why.
- In reality, most of the capital is owned by the firms (not rented).
- Capital is a stock variable that builds and depreciates over time.
- We will assume that this investment is made by the firm in a competitive market.

Model: capital evolution

- The evolution of capital is characterized by

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{2}$$

  where
  - K_t stock of capital at time t
  - δ depreciation rate
  - I_t Investment in period t
- The level of capital in period t to is a “state” variable (i.e., given)
- This will imply that we can only affect capital (tomorrow) trough investment today.
- When investment equalizes depreciation, capital does not change: $\Delta K = 0 \iff I_t = \delta K_t$

Model

- Since investment today defines the stock of capital tomorrow, the problem of the firm becomes dynamic.
- Assume the price of a unit of investment good is $p_{K,t}$ and firms discounting time by $\frac{1}{1+r}$ (why does this make sense?)
- The period t profits are:

$$\Pi_t = A_t F(K_t, X_t) - p_{X,t} X_t - p_{K,t} I_t$$

where $A_t F(K_t, X_t)$ is output, $p_{X,t} X_t$ is the expenditure in other factors, and I_t is investment.

Given the initial level of capital and total factor productivity, the problem of the firm is

$$\max_{\{X_t, K_{t+1}, I_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t [A_t F(K_t, X_t) - p_{X,t} X_t - p_{K,t} I_t]$$

subject to:

$$K_{t+1} = (1-\delta)K_t + I_t \quad \forall\, t$$

$$K_0 \text{ given}$$

Model: solution to the firm's problem

Replace investment, given by the capital evolution constraint, into the discounted profits and take first order conditions with respect to other factors and capital in next period to get

$$\frac{\partial(\cdot)}{\partial X_t} = 0 \implies p_{x,t} = A_t F_X(K_t, X_t) \tag{3}$$

$$\frac{\partial(\cdot)}{\partial K_{t+1}} = 0 \implies p_{k,t} = \frac{1}{1+r}\left[A_{t+1} F_K(K_{t+1}, X_{t+1}) + p_{k,t+1}(1-\delta)\right] \tag{4}$$

- As usual, marginal profits equalizes marginal cost
- The marginal cost of increasing capital tomorrow is the price of investing today.
- The marginal profit — discounted because it happens tomorrow— has two parts: marginal revenue of the increase in capital and its re-selling price after it's used (i.e., depreciated)

Model: user cost of capital

- Re-arrange (4) to get

$$p_{k,t}(r+\delta) - \Delta p_{k,t+1}(1-\delta) = A_{t+1} F_K(K_{t+1}, X_{t+1})$$

- By comparing the first order condition of the rental model in (1) with the condition above, this defines the user cost of capital $r_{K,t}$ as:

$$r_{K,t} = p_{k,t-1}(r+\delta) - \Delta p_{k,t}(1-\delta)$$

User cost of capital

The user cost of capital is the cost associated with the use of a unit of capital for one period.

Example

User cost of capital

- Assume you own a business that uses trucks
- The price of new trucks today is \$ 50K. You know that tomorrow the price will fall to \$ 45.
- To buy one truck you borrow from a commercial bank which will charge a 10% interest rate.
- Once you finish using it, the truck would have depreciated 5%.
- The total cost of using one truck for one period is:

$$r_k = 50 * (1 + 0.1) - 45 * (1 - 0.05) = 12.25$$

- What would happen if the rental price of trucks is \$7K?

Model: desired capital stock

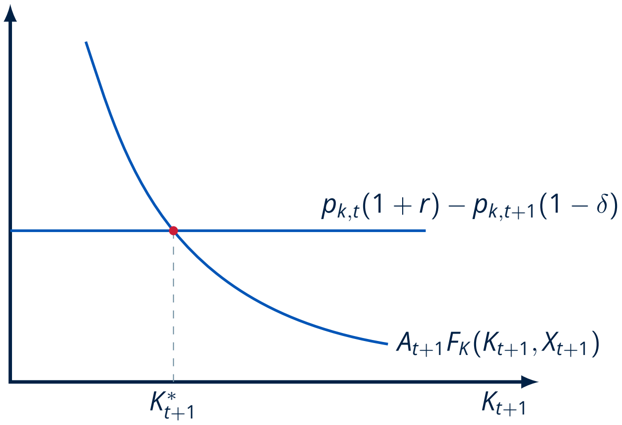
- Remember that the capital stock today is not part of the choices of the firm today!
- The first order condition with respect future capital yields the demand for future capital K^*_{t+1}.

$$A_{t+1} F_K(K^*_{t+1}, X_{t+1}) \quad = \quad p_{k,t}(1+r) - p_{k,t+1}(1-\delta)$$

- Since the marginal productivity of capital decreases as capital increases, the left hand side of the previous equation is a downward sloped curve.
- The right hand side does not depend on capital, and therefore is an horizontal line.

Model: desired capital stock

Figure 4: Optimal choice of capital



- To get tomorrow's desired level of capital K^*_{t+1}, the firm needs to invest I^*_t today. Use (2) to get

$$I^*_t = K^*_{t+1} - (1-\delta)K_t \tag{5}$$

- Whatever makes change the optimal capital stock, changes in the same proportion the optimal investment —Except for depreciation δ.

- The optimal investment allows us to achieve the optimal capital stock in one period (i.e., no dynamics beyond that horizon)

Model: comparative statics

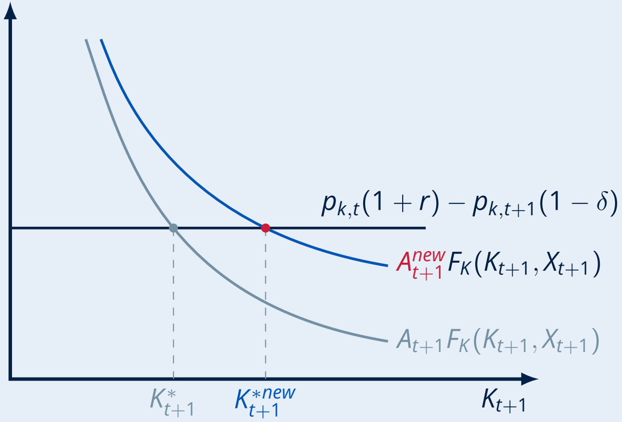
Keep everything else constant and increase:

- A_{t+1}: MPK increases. To take advantage of higher productivity, increase K_{t+1} and invest more.
- X_{t+1}: Similar to A_{t+1} because of complementarity.
- $p_{k,t+1}$: increase in the future value of capital, increase K_{t+1}.
- δ: decrease in the re-selling value of capital, decrease K_{t+1}.
- A_t or X_t: no effect!

Comparative statics: example

Increase in A_{t+1}

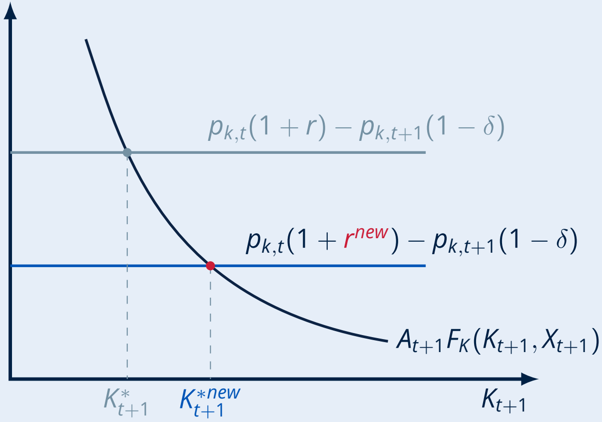
Figure 5: Capital choice, increase in A_{t+1}



Comparative statics: example

Decrease in r

Figure 6: Capital choice, decrease in r



Model: Adjustment costs

- So far investment just look forward one period (any news from $t + j$ with $j > 1$ would not affect investment)
- Some capital can be constructed easily, but other capital may take years to put in place
- Investment needed to reach the desired capital stock might spread out over several periods.
- How can we model this feature? Adjustment costs
  - If firms face a *convex* cost of changing their level of capital, they would spread that change over time.
  - Capital will converge (at a decreasing rate) to its desired level on the long-run.
  - The anticipation of shocks in the future induce changes in the present.

Model: Tobin's Q

- Firms change investment in the same direction as the stock market: Q theory of investment , Tobin (1969)
- If market value $>$ replacement cost, then firm should increase its capacity.
- Tobin's average $Q = \dfrac{\text{Market Value of Firm}}{\text{Replacement cost of capital}}$
  - If $Q < 1$, invest less
  - If $Q > 1$, invest more

Model: Tobin's Q

How would one measure Q in the data?

- Typically this is done by using the value of the stock market in the numerator and data on corporate net worth in the denominator.

Average Q

- Suppose that a company is in the crane business.
- Suppose that it owns 100 cranes, which cost 100 each to replace. This means its replacement cost is $10,000$
- Suppose it has $1,000$ outstanding shares of stock, valued at 15 per share. The market value of the firm is thus $15,000$
- Tobin's Q is $1.5 \implies$ The firm should evidently buy more cranes?

- Average vs marginal decisions and optimality, Hayashi (1982)

Efficient Markets Hypothesis (EMH)

Efficient Markets Hypothesis

- Each stock price reflects all available information about the stock.
- The market price of a company's stock is the fully rational valuation of the company, given current information about the company's business prospects.

Implies that stock prices should follow a random walk (be unpredictable), and should only change as new information arrives.

Keynes beauty contest and EMH

- Newspaper contest: participant wins a prize if he/she picks the women most frequently selected by others as most beautiful.
- Keynes: stock prices reflect people's views about what other people think will happen to stock prices.
- Stock prices reflect irrational waves of pessimism/optimism.

Model: Tobin's Q and adjustment costs

- Assume that every time that firms want to adjust its scale, has to pay an "adjustment cost"

$$AC(I_t, K_t) = \frac{\phi}{2}\left(I_t - \delta K_t\right)^2$$

- Assume the profits to be

$$\Pi_t = A_t F(K_t, X_t) - p_{x,t} X_t - I_t - AC_t$$

- The problem of the firm is

$$\max_{\{X_t, I_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \Pi_t$$

subject to

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + I_t \\ \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T q_T K_{T+1} &= 0 \end{aligned}$$

Model: Tobin's Q and adjustment costs

- Solve as Lagrangian
- Denote q_t as the Lagrange multiplier of restriction in t

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \Pi_t + q_t \left[(1-\delta) K_t + I_t - K_{t+1} \right]$$

where

$$\Pi_t = A_t F(K_t, X_t) - p_{x,t} X_t - I_t - \frac{\phi}{2} \left(I_t - \delta K_t \right)^2$$

- What does represent q_t? (the marginal value of an additional unit of capital)

Model: Tobin's Q and adjustment costs

- The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial X_t} = 0 \implies \quad p_{x,t} = A_t F_X(K_t, X_t) \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial q_t} = 0 \implies \quad K_{t+1} = (1-\delta)K_t + I_t \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} = 0 \implies \quad q_t = 1 + \phi\left(I_t - \delta K_t\right) \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \implies \quad q_t = \frac{1}{1+r}\left[A_{t+1} F_K(K_{t+1}, X_{t+1}) + \delta\phi(I_{t+1} - \delta K_{t+1}) + q_{t+1}(1-\delta)\right] \tag{9}$$

Model: Tobin's Q and adjustment costs

- From (7) and (8) we get

$$\Delta K = K_{t+1} - K_t = \frac{q_t - 1}{\phi} \tag{10}$$

- From (8) and (9) we get

$$\Delta q = q_{t+1} - q_t = rq_t - A_{t+1} F_k(K_{t+1}, X_{t+1}) + \delta \tag{11}$$

- This is a system of difference equations that dictates the path of q_t and K_t for any given K_0 and q_0

Model: steady state

- In steady state $\Delta K = \Delta q = 0$
- Replacing this in (10) and (11) we get

$$\Delta K = 0 \implies q^{ss} = 1 \tag{12}$$

$$\Delta q = 0 \implies q^{ss} = \frac{AF_k\left(K^{ss}, X\right) - \delta}{r} \tag{13}$$

- To analyze the path of q and K we will plot $\Delta K = 0$ and $\Delta q = 0$ in a graph with q and K as coordinates and see what happens if we are outside of the steady state.

Model: Q phase diagram

Figure 7: $\Delta K = 0$

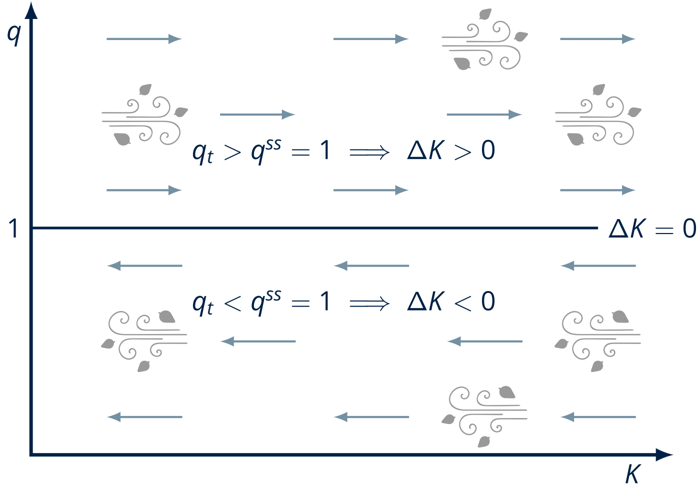


Figure 8: $\Delta q = 0$

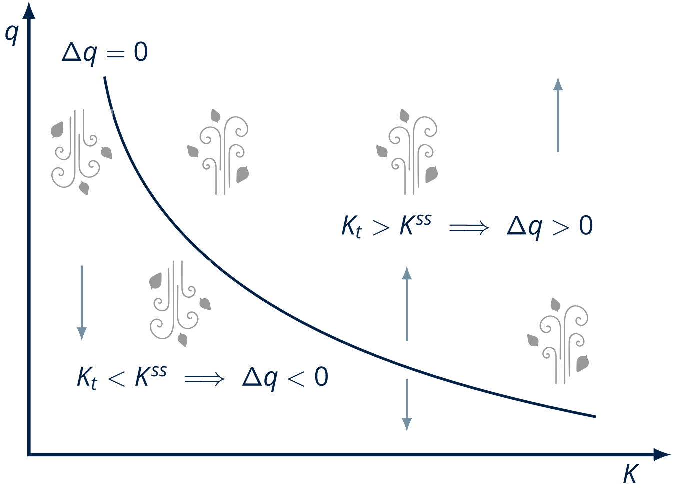
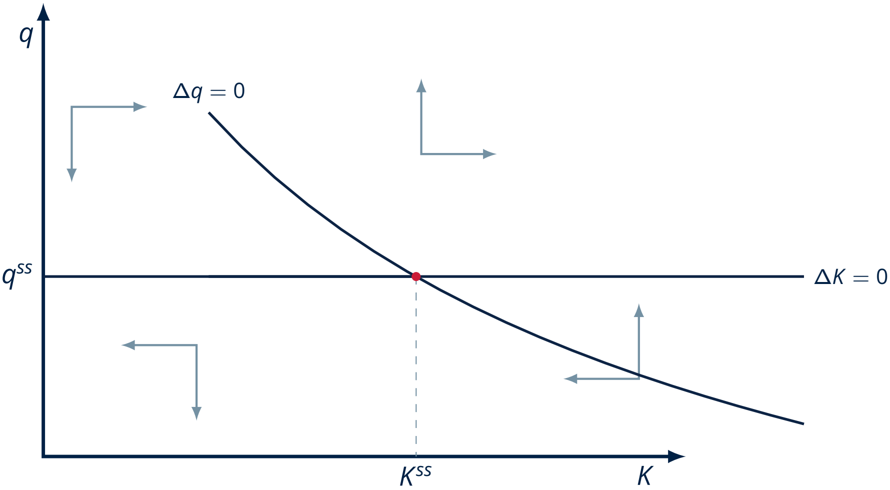


Figure 9: Phase diagram



Model: transversality condition and convergence

- Note that when standing to the north east of the steady state the economy diverges. The "winds" will blow capital and q towards infinity
- In general all paths —except one— will diverge.
- The transversality condition rules out diverging paths.
- The Saddle Point Path is the only path that leads to the steady state q and capital stock.
- Given K_0, there's only one q_0 that leads to the steady state. If there's no anticipated changes, that q_0 must be in the SPP.

Dynamics without transversality condition

Figure 10: K and q dynamics

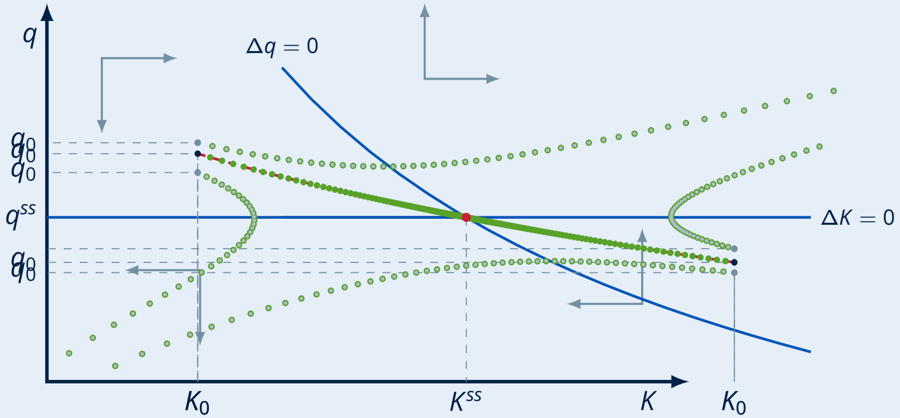
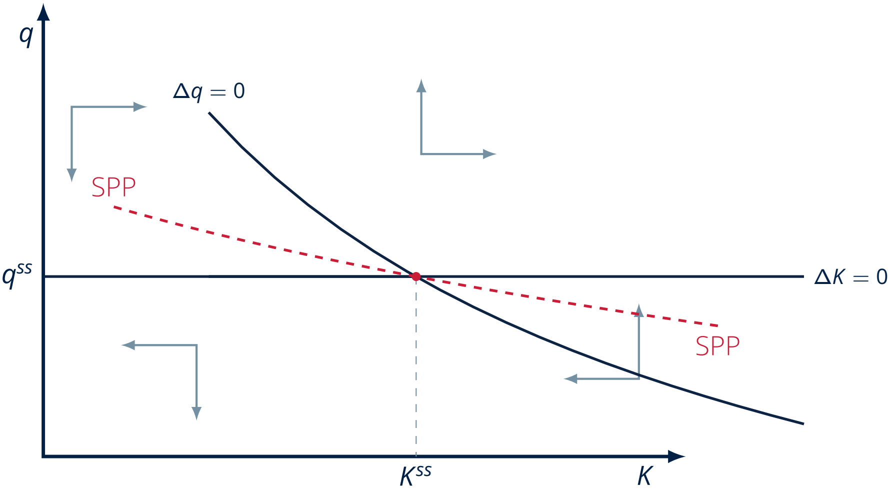


Figure 11: Saddle point path

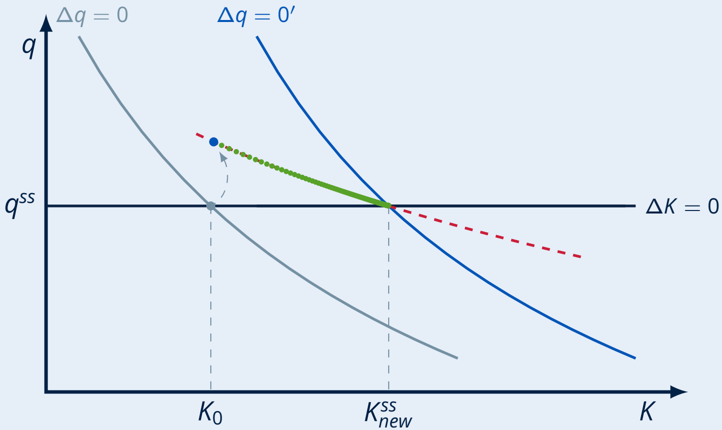


Unanticipated increase in TFP

- Assume the economy to be in steady state.
- At time $t = 0$ the total factor productivity increases permanently from A to $A' > A$
- The new steady state capital is higher:
  - The locus $\Delta K = 0$ is not affected
  - $\Delta q = 0$ shifts up.
- The new saddle point path of the economy is above the current q, so q_0 must jump up.
- As the time passes, q_t will get closer to 1 and K_t will increase to converge to its new steady state.

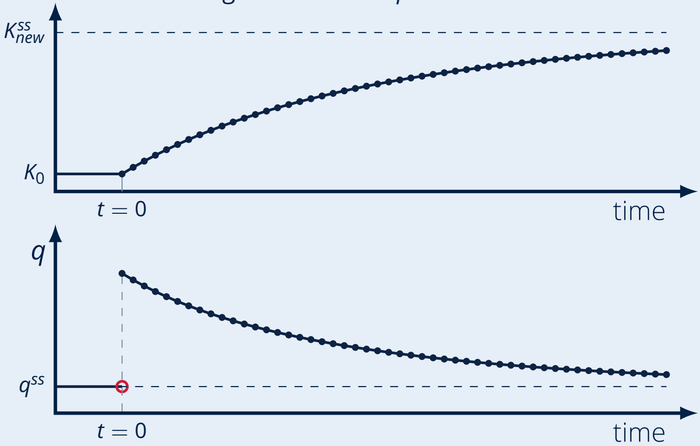
Unanticipated increase in TFP (cont)

Figure 12: TFP increase, unanticipated



Unanticipated increase in TFP (cont)

Figure 13: K and q over time

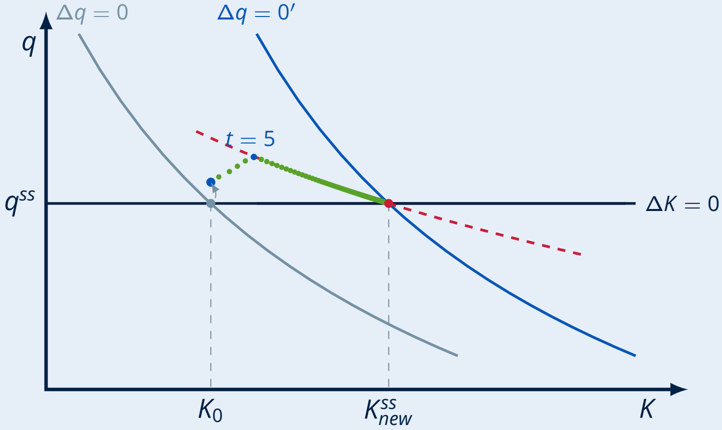


Anticipated increase in TFP

- At time $t = 0$ the private sector forecast a permanent total factor increase, from A to $A' > A$, but in 5 periods from now
- The new steady state capital is higher.
- For the investors is optimal to be in the new SSP when the change happens.
- Since the economy hasn't change yet, the "winds" that rule the economy are the same. To get out of the current steady state q jumps up, but less than the jump with an unanticipated shock.
- Capital and q drift up until $t = 5$, when the economy is at the SSP. This is important, because both change before the realization of ΔA
- As the time passes, q_t will get closer to 1 and K_t will increase to converge to its new steady state.

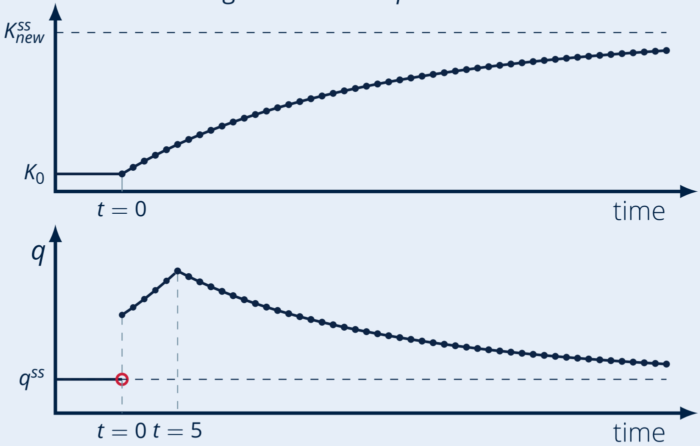
Anticipated increase in TFP (cont)

Figure 14: K and q over time



Anticipated increase in TFP (cont)

Figure 15: K and q over time



Model: more examples

- There are several cases that we could analyze for changes in different variables, by combining anticipation and transitoriness.
- It is also worth to see if there's any difference when we change the anticipation or persistence horizon. (There is!)
- A good example — that might or might not appear in the exam— would be to see the dynamics when firms expect a change in $t \neq 0$, but when the time comes that expectation does not materializes.

Summary and comments

Summary and comments

- A profit maximizer firm will choose capital equalizing marginal cost to marginal revenues.
- The marginal productivity of capital defines real marginal revenue.
- By definition, the user cost corresponds to the marginal cost of using capital in one period. This cost will be composed of three elements: interest, depreciation and capital loss/gain.
- All the elements that change the desired level of capital also change the level of investment since both are related through the law of motion of capital (5).
- Increases in r lead to a decrease in the desired level of capital and therefore a decrease in investment.

Summary and comments

- When firms can invest as much as they want, the relevant horizon — in our model— is one period.
- Information about the future ($t > 2$) is irrelevant for investment unless we introduce convex adjustment costs.
- With convex adjustment costs the firm plans ahead in order to smooth capital (and q) as much as possible
- This will imply that future changes in productivity, for example, result in more investment today.

Summary and comments

- In general the data supports the user cost of capital in the long run.
- In the short run, other variables seem to explain more of the variation of investment.
- Even with data refinements and more accurate measures of Q.
- Micro data shows that the vast majority of individual firms investment is infrequent and lumpy.
- The neoclassical model we saw assumes a firm can borrow as much as it needs. Firms often find financial constraints.
- Caballero (1999): Informational problems, sunk costs, investment irresistibility, financial constraints, etc. $\implies$ strategic interactions and opportunistic behavior that affect investment

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