



Labour Market

ECON 302, Intermediate Macroeconomics

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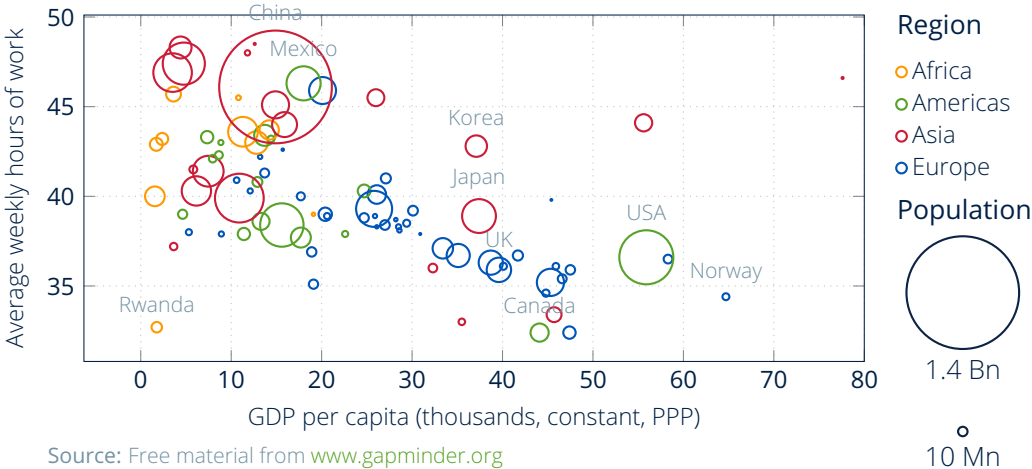
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Introduction

- In this class we will analyze the labor market.
- This is a key market for the aggregate supply (labor is an important production input).
- The model we will see in will be the “standard” competitive market.
- Many of the assumption we will do will end up being to restrictive if we want to replicate the short run data.
- Nonetheless it will show some of the mechanisms through which the labor input and wages are determined.

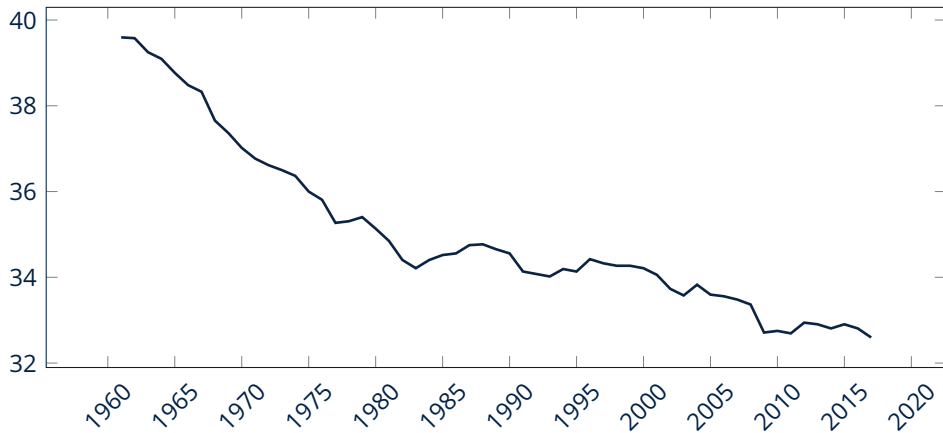
- In general we observe that richer countries work less hours per week compared to poorer ones.
- This is also true across time. As countries get richer, they tend to work less hours.
- This is a long term story. In the short run income and labor input seem to be positively correlated.
- One characteristic feature of the labor market is that it never clears: the unemployment rate is, and has been always, positive.
- Unemployment tend to increase in recessions and decrease during booms.

Figure 1: Average weekly hours vs income (2017)



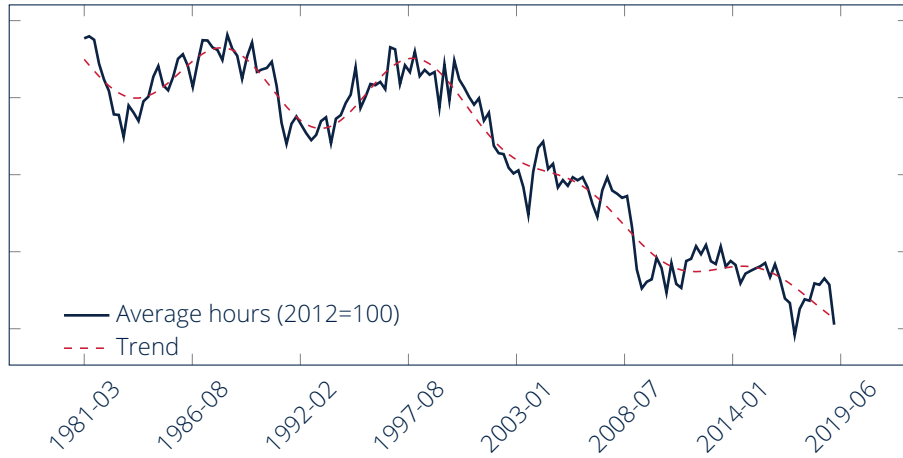
Source: Free material from www.gapminder.org

Figure 2: Average weekly hours actually worked per worker, Canada



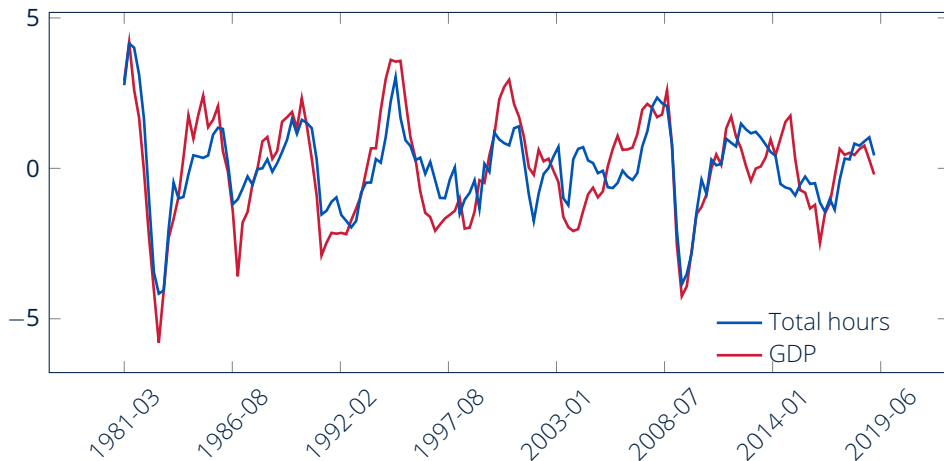
Source: OECD.Stat

Figure 3: Average weekly hours index, Canada



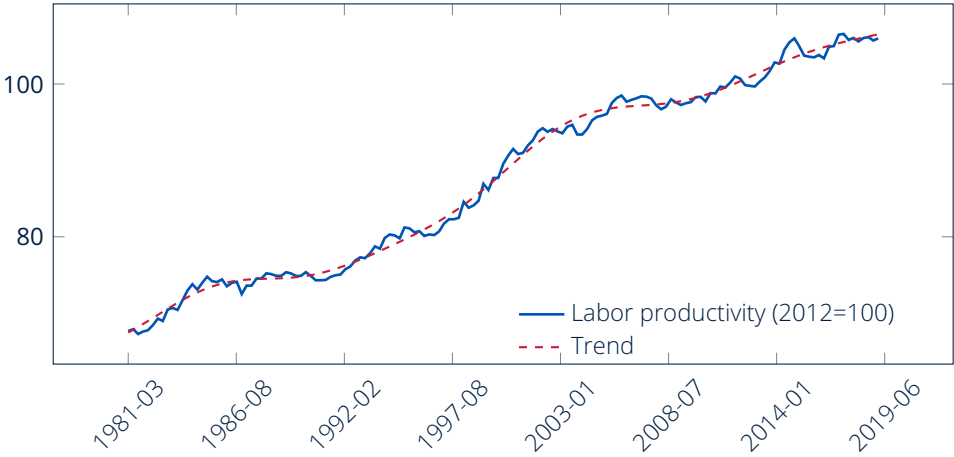
Source: Statistics Canada and own calculations

Figure 4: Cyclical components of labor input and GDP, Canada



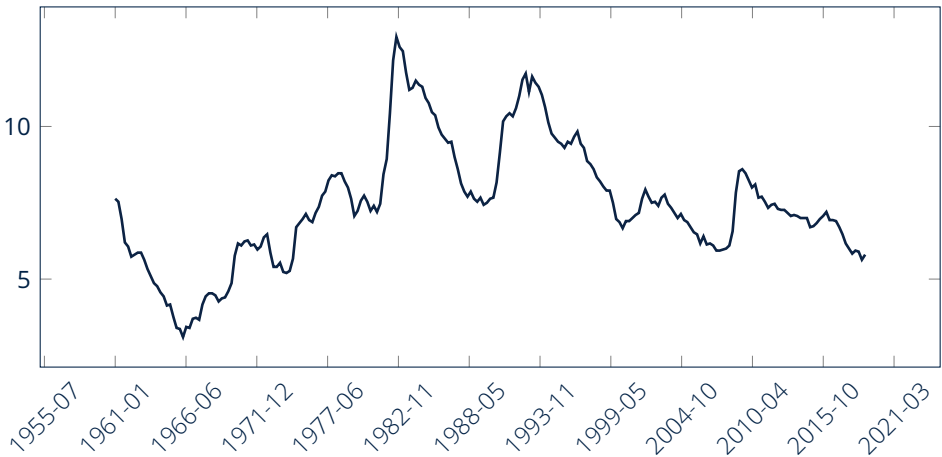
Source: Statistics Canada and own calculations

Figure 5: Labor productivity and its trend, Canada



Source: Statistics Canada and own calculations

Figure 6: Unemployment rate (%), Canada



Source: OECD.Stat

- At the end of this lecture you should understand what are the factors that affect the demand and supply of labor.
- Particularly, it should be clear why the labor supply might backward bend (understand the income and substitution effect)
- In this class we will explain some of the assumptions that lead to observe unemployment. You should have a notion on why this might happen and how it relates to the “real world”.
- You should be able to define “natural” unemployment and how it relates to the full employment line.

A basic exposition of part of the material in this lecture can be found in

- [Abel et al. \(2018, Chapter 3.2-3.5\)](#)
- [Mankiw \(2009, Chapter 3.2\)](#)

For a more complete version look (recommended)

- [Borjas \(2019, Chapter 2\)](#) ([available in this link](#))

Short math review

- Assume you have a multivariate function $F(x_1, \dots, x_N)$ you would like to maximize.
- Let $\sum_{n=1}^N a_n x_n = A$ and $\sum_{n=1}^N b_n x_n \leq B$ two constraints on x_n
- The problem you would like to solve is

$$\max_{\{x_n\}_{n=1}^N} F(x_1, \dots, x_N)$$

subject to

$$\sum_{n=1}^N a_n x_n = A \quad \text{and} \quad \sum_{n=1}^N b_n x_n \leq B$$

- The Lagrangean of the previous problem is

$$\begin{aligned}\mathcal{L}(x_1, \dots, x_N, \lambda_1, \lambda_2) = & F(x_1, \dots, x_N) \\ & - \lambda_1 \left[\sum_{n=1}^N a_n x_n - A \right] - \lambda_2 \left[\sum_{n=1}^N b_n x_n - B \right]\end{aligned}$$

- The first order conditions are (Karush-Khun-Tucker):

$$\begin{aligned}\frac{\partial \mathcal{L}(x_1, \dots, x_N, \lambda_1, \lambda_2)}{\partial x_n} &= 0 \quad \forall n \in \{1, \dots, N\} \\ \lambda_i \frac{\partial \mathcal{L}(x_1, \dots, x_N, \lambda_1, \lambda_2)}{\partial \lambda_i} &= 0 \quad \forall i \in \{1, 2\}\end{aligned}$$

- The solution is found by solving the system of equations ($N + 2$ equations and unknowns) given by the first order conditions

Labor demand

Who demands labor? Who supplies labor?

- Firms choose labor demand optimally
- Households choose labor supply optimally
- Government may affect optimal choices using various policy tools (taxes, transfers)
- The equilibrium objects are the labor input (quantity) and the real wage (price).

To develop a model we need to clarify what are the assumptions and how a change in them would change the outcome. For our basic model we will assume:

- All workers and firms are identical
- A single good is produced.
- Firms and workers are price takers (competitive market)
- The economy is in the short run (i.e., the level of capital exogenously given $K = \bar{K}$)
- No frictions in the labor market. (e.g., price or wage rigidities)
- Firms use production technology as in the production model.
- Labor demand maximize firms profits.
- Labor supply maximize workers utility.

- In a competitive market firms don't choose the price.
- If the good and labor markets are competitive, then firms take the good price (P) and a nominal wage (W) as given.
- Given technology , capital stock, and wages how much labor should the firm demand?
- The firm that maximizes profits will demand labor until its marginal product equalizes real wages (i.e. marginal benefit = marginal cost)

- The firm problem consist in maximize profits given prices, capital level and technology

$$\max_{N^d} P \cdot AF(\bar{K}, N^d) - W \cdot N^d$$

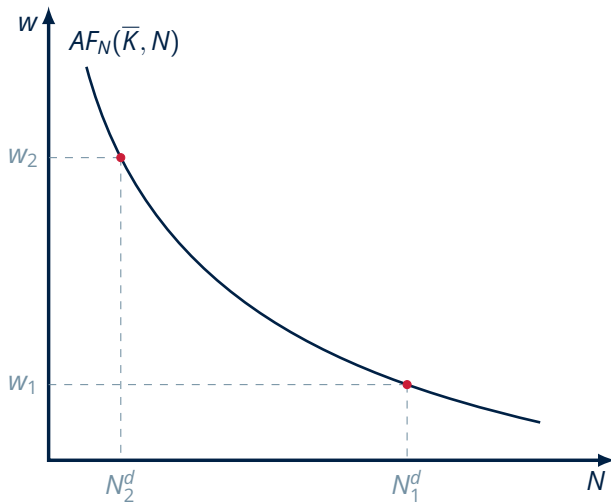
- Where N^d is the labor demand, $\bar{K}$ is the stock of capital, A is the TFP, P is the good price, and W is the nominal wage.
- The first order condition of the firm (with respect to labor) is

$$W = P \cdot AF_N(\bar{K}, N^d)$$

- Denote $w = W/P$ as real wages, then the FOC implies

$$w = AF_N(\bar{K}, N^d) \tag{1}$$

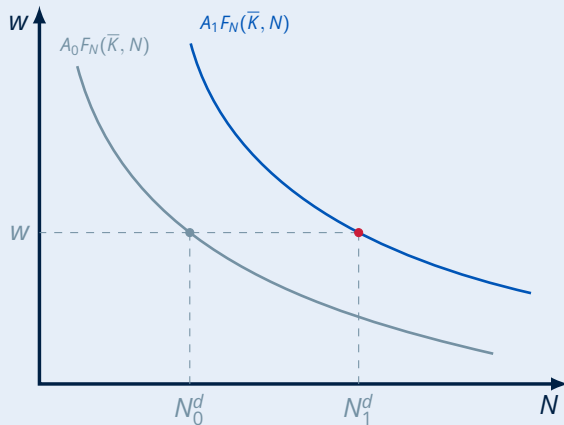
Figure 7: Labor demand



- The demand curve *does not* shift as a result of a change in the real wage:
 - Real wage is an endogenous variable
 - If the real wage changes, the firm responds by *moving along* the curve
- “Curve Shifters” are exogenous forces that increase (or decrease) the marginal productivity of labor (MPN)
 - The firm would like to hire more (or less) given the real wage
 - What are the curve shifters in our model?
 - An increase in either A or $\bar{K}$ increases MPN.

Increase in TFP and the change in the labor demand

Figure 8: Increase in TFP



Labor supply

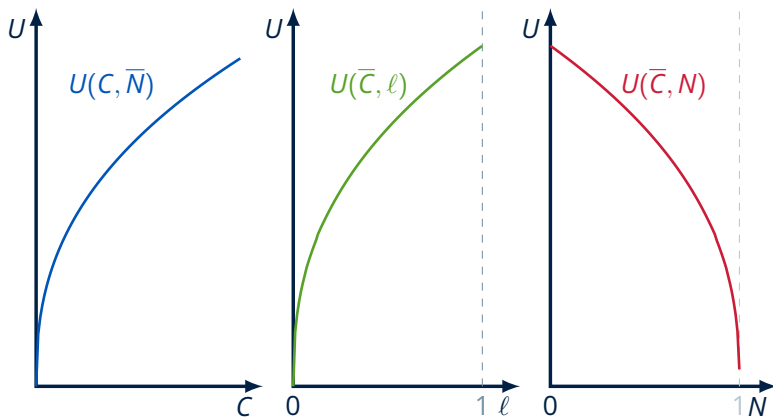
- Given technology, capital stock, and wages, how much labor should the household supply?
- Further assumptions about household preferences and constraints:
 - Households / Consumers / Workers (all identical in our model) derive utility from consumption (C) and leisure (ℓ)
 - Endowed with 1 unit of time for work/leisure ($\ell = 1 - N$)
 - More consumption is better, and last unit is “less satisfying” than the one before
 - More leisure is better, and last unit is “less satisfying” than the one before
 - More labor is worse, and last unit is “more painful” than the one before

- A utility function is an object that we can use to characterize preferences.

$$U(C, N) = U(C, 1 - \ell)$$

- Our assumptions in math mode:
 - More consumption is better: $U_C > 0$
 - The last unit is less satisfying: $U_{CC} < 0$
 - More leisure is better: $U_\ell > 0$
 - The last unit of leisure is less satisfying: $U_{\ell\ell} < 0$
 - More labor is worse: $U_N < 0$
 - The last unit is more painful: $U_{NN} < 0$

Figure 9: Utility function shape



Utility function: Cobb-Douglas

$$U(C, N) = C^\gamma (1 - N)^{1-\gamma}$$

Utility function: Additively separable (CRRA)

$$U(C, N) = \frac{C^{1-\sigma} - 1}{1-\sigma} - \xi \frac{N^{1+\eta} - 1}{1+\eta}$$

Utility function: Inelastic

$$U(C, N) = \frac{C^{1-\sigma} - 1}{1-\sigma}$$

How much do you think the household will work and consume given the previous preferences?

With no constraints: don't work and consume infinity!

- Assume the budget constraint to be

$$(1 + \tau_c)C \leq (1 - \tau_n)wN + B$$

- τ_c is a proportional tax on consumption.
- τ_n is a proportional labor income tax.
- B is some initial wealth plus lump-sum transfers.
- We can assume that the constraint holds with equality (the restriction will always bind)

- The household problem

$$\begin{aligned} \max_{C, N} & U(C, N) \\ \text{s.t.} \quad & (1 + \tau_c)C = (1 - \tau_n)wN + B \\ & N \in [0, 1] \end{aligned}$$

- The solution (interior)

$$-U_N(C, N) = \frac{1 - \tau_n}{1 + \tau_c} w U_C(C, N) \quad (2)$$

$$(1 + \tau_c)C = (1 - \tau_n)wN + B \quad (3)$$

- 2 equations and 2 unknowns: C and N

What happens if we start from some allocation $\tilde{C}, \tilde{N}$, and marginally increase N ?

- The left hand side of (2) is the “marginal pain” or cost of the extra unit of labor.
- What is the marginal benefit? working an extra hour pays $(1 - \tau_n)w$
- We use the extra earnings to consume more: $\frac{1 - \tau_n}{1 + \tau_c}w$
- The marginal benefit of each extra unit of consumption is U_C , therefore the “total” marginal benefit is the right-hand-side of (2)
- We will refer to (2) as the [static first order condition](#)

Figure 10: Optimal consumption-Leisure decision

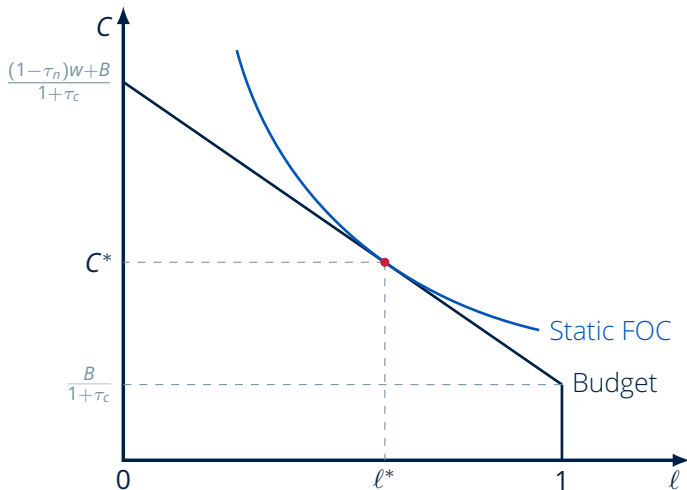
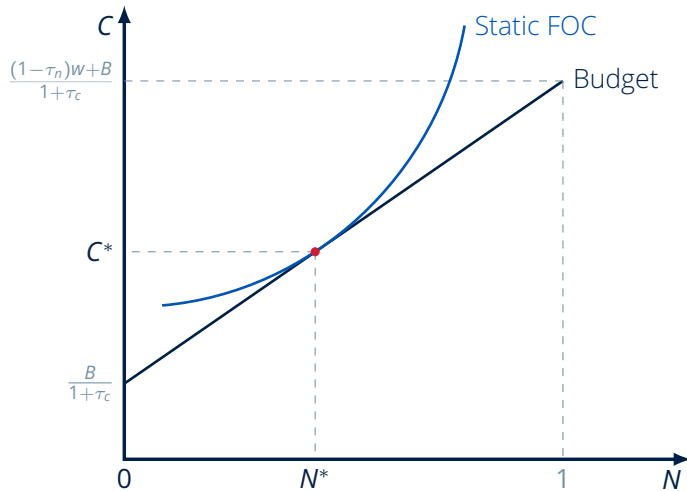


Figure 11: Optimal consumption-Labor decision



Increase in wages

- Assume you work 20 hours per week and your hourly wage jumps up by \$10. Would you work more or less?
 - Now assume that your employer will pay you 1 million dollars per hour. Would you work more or less?
-
- Wage affects the budget constraint and the static first order condition
 - On one hand, higher wage makes you richer: you can work less (increase leisure) and keep your level of consumption (Income effect, IE)
 - On the other, higher wage makes leisure more expensive or working more attractive (Substitution effect, SE).

Labor supply: substitution and income effects

- Equations (2) and (3) define the *uncompensated* labor supply $N^*(w, B)$
- Minimizing $(1 + \tau_c)c - (1 - \tau_n)wN$ subject to $U(C, N) = \bar{U}$, yields the *compensated* supply $N^c(w, \bar{U})$
- The *Slutsky's* equation define the wage elasticity of labor:

$$\frac{\partial \log(N)}{\partial \log(w)} = \underbrace{\frac{\partial N^c(w, \bar{U})}{\partial w} \frac{w}{N}}_{\text{substitution effect}} + \underbrace{\frac{\partial N^*(w, B)}{\partial B} w}_{\text{income effect}} \quad (4)$$

- If leisure is a *normal good* there is a chance that the income effect dominates the substitution effect. This would yield a backward bending labor supply

Additively separable preferences

- Assume the household has additively separable preferences, and no taxes (budget: $C = wN + B$)

$$U(C, N) = \frac{C^{1-\sigma} - 1}{1-\sigma} - \frac{N^{1+\eta} - 1}{1+\eta}$$

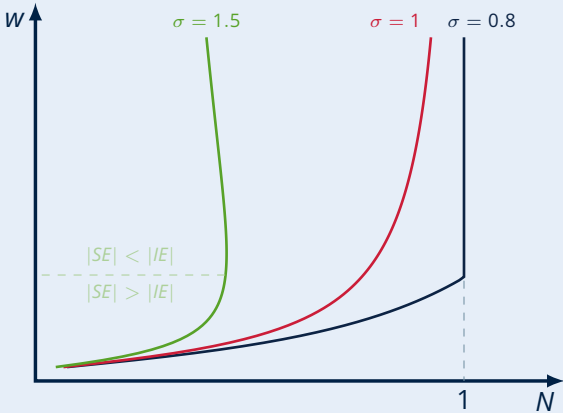
- The labor supply of the household is given by (board)

$$N^{-\eta/\sigma} w^{1/\sigma} - wN = B$$

- Assume further that $B = 1$ and $\eta = 0.5$
- If $\sigma < 1$ then the labor supply is upward sloped.
- If $\sigma > 1$ the labor supply backward bends at high wages.
- If $\sigma = 1$ the labor supply converges to $N = 1$ at high wages.

Additively separable preferences (cont.)

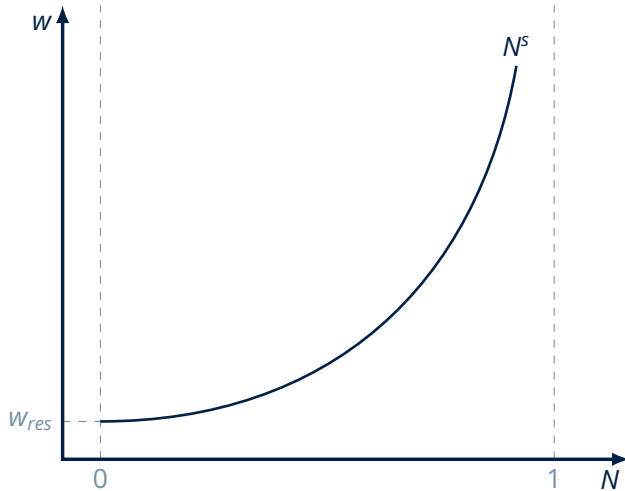
Figure 12: Labor supply



Increase in wages

- Assume your employer increase your wage to 1 million dollars per hour. Would you work more or less?
- We are looking to build a framework to analyze short run fluctuations.
- We will assume that the substitution effect dominates the income effect.
- This will imply that an increase in wage will lead to more hours worked, at least in the short run.

Figure 13: Labor supply



- How would the household react if, for example, future expected wages fall?
- We cannot answer that question: our model is static. Don't worry, we will keep it that way.
- If we solve the consumption model with endogenous labor supply ($y_t = w_t N_t$ in the budget constraint) we would get as solution the Euler equation, the static FOC, and the life-time budget constraint.
- We can sneak in part of the dynamics by interpreting B in a richer way.

- Think about B as the present value of life-time resources (PVLR)

$$B = W_0 + \sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^t (w_t N_t - T_t) \quad (5)$$

- Where W_0 is some initial wealth, $w_t N_t$ is the labor income in period t , and T_t are taxes/transfers.
- This is a broader interpretation of wealth.
- Lower wages in the future implies lower PVLR $\implies$ lower B .

- A change in B affects only the budget constraint.
- Higher B implies that we can consume more
- But we can also work less to keep the level of consumption constant.
- The recommendation given by the static FOC is to do both: Increase C and decrease N
- An increase in B shift the labor supply to the left.

- Questions to answer when thinking of taxes
- How taxes are collected?
 - Lump-sum vs proportional
 - Taxes on labor or consumption
- How is the tax revenue used?
 - Spending in goods and services without effects in the household's utility.
 - Spending with utility implications to the household
 - Rebates
- How permanent the policy is?
 - In our model permanent changes, might have PVLR effects.
- Different answers trigger mechanisms that might affect the labor supply in different ways

If taxes are lump-sum:

- Causes a negative wealth effect, as $T_t = \bar{T}$ affects PVLR.
- The more permanent the tax is, the stronger the effect.
- An increase in lump-sum taxes shifts the labor supply curve to the right.
- If the tax revenue is rebated to consumers in the form of transfers then the tax change may be “PVLR neutral” (no changes on labor supply)

When taxes are proportional (τ_c, τ_n) we affect the consumption-leisure optimal proportion, causing a distortion of the labor supply

- static FOC: $-U_N = \frac{1 - \tau_n}{1 + \tau_c} w U_C$
- If $\tau_c = \tau_n = 0$ then $-U_N = w U_C$ and the economy is considered non-distorted
- If we increase τ_n (or τ_c), keeping everything else equal, the value of the RHS declines.
- Decrease N and C until LHS=RHS.
- Increasing tax rates would increase the distortion: left-shift of the supply curve
- If the tax increase is persistent it will cause a negative PVLR effect, shifting the supply to the right.

Some of the variables that can cause a shift of the labor supply:

- Population/participation: If more people participate in the labor market we add their labor supply horizontally and, therefore, the supply curve shifts to the right
- PVLR/Wealth: if **B** decrease the labor supply curve shifts to the right.
- Taxes: mixed effects.

Shift vs move along the labor supply

Changes in the **current level** of the real wage cause a movement along the labor supply curve. However, **future** wage changes may have an effect on PVLR, and therefore may cause a shift of the labor supply curve

Labor market equilibrium

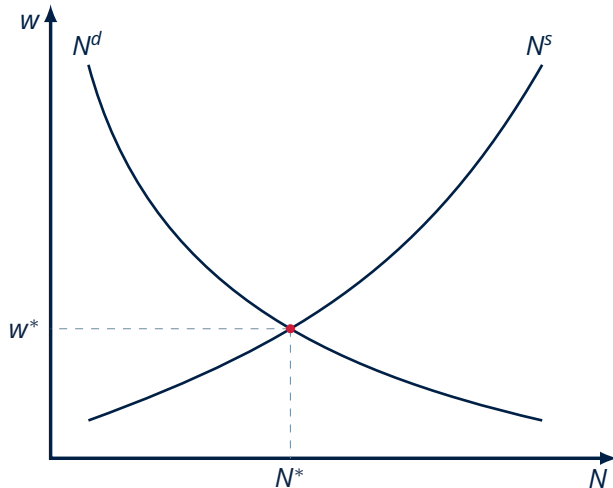
Equilibrium in the labor market is defined as a real wage w^* , and labor input N^* such that:

- Households labor supply choice is optimal (i.e. satisfy the static FOC and budget)
- Firms labor demand choice maximizes profits (i.e. $MPN = w$)
- The market clears (Supply=Demand)

Note that when the labor market is in equilibrium, the level of output is determined by

$$Y^* = AF(N^*, \bar{K})$$

Figure 14: Labor market equilibrium

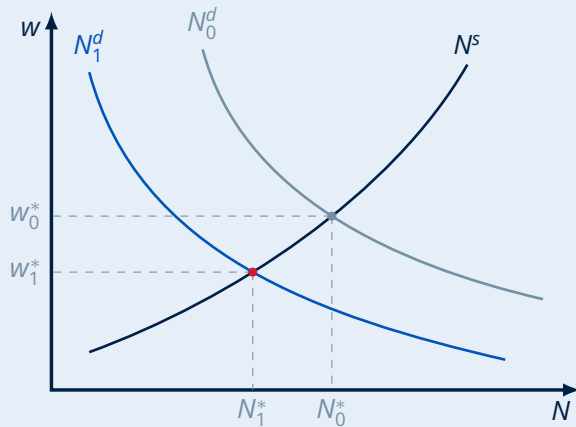


A negative temporary shock to TFP

- With lower A , the marginal product of labor is lower for any given level of N .
- The labor demand curve shifts down.
- Labor supply curve does not shift.
- If wage does not adjust there is an excess supply. Wage needs to drop in order to clear the market.
- This shock is temporary so there is no PVLR effect.
- A new equilibrium with lower N and lower w
- Output declines (TFP and N fall)
- Average labor productivity (Y/N) declines (this is not trivial)

A negative temporary shock to TFP (cont)

Figure 15: Labor supply shift due to a negative TFP shock

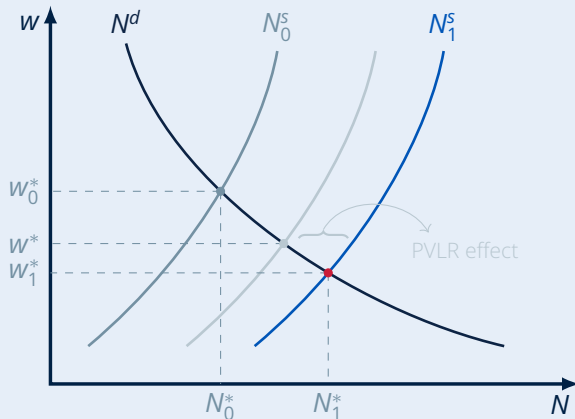


A permanent increase in population

- This is a direct shock to labor supply: N^S shifts to the right
- No movement of the demand.
- The equilibrium will require higher N , lower w
- Because this is permanent, people will expect lower wages in the future, thus lower PVLR. This further shift the supply curve to the right.
- For the same level of capital and TFP, we produce more output.
- Labor productivity is lower.

A permanent increase in population (cont)

Figure 16: Labor supply shift due to a permanent increase in population



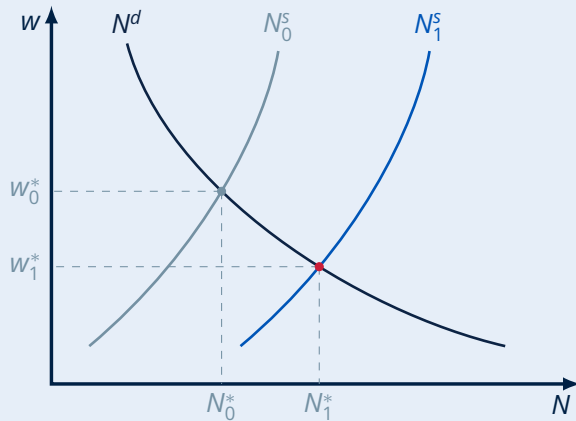
A tax and spending reform

Assume a permanent decrease of the labor income tax τ_N , coupled with a decrease in transfers by the same total amount

- No shift of demand
- Distortion effect becomes smaller, implies supply shifts to the right
- The lower taxes paid have a positive effect on PVLR, but the lower transfers received cancel this effect: no PVLR effect
- New equilibrium with higher N , lower w
- Production function implies more output
- Labor productivity is lower

A tax and spending reform (cont)

Figure 17: Labor supply shift due to decrease in labor income tax



Unemployment and the FE line

Labor force

The labor force, also known as currently active population, comprises all persons who fulfill the requirements for inclusion among the employed (civilian employment plus the armed forces) or the unemployed.

Employed

Employed are those who work for pay or profit for at least one hour a week, or who have a job but are temporarily not at work due to illness, leave or industrial action.

Unemployed

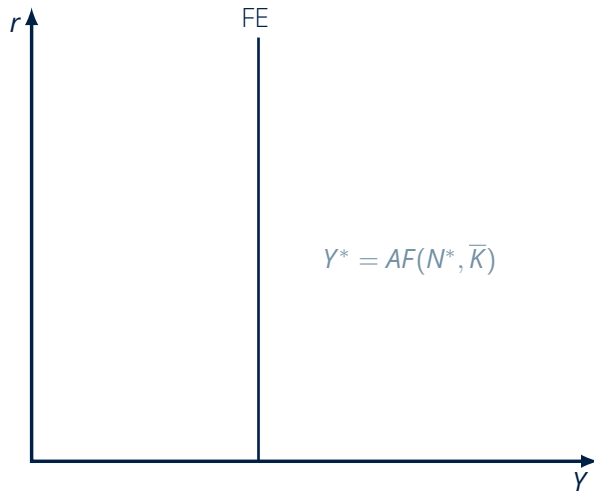
Unemployed are those without work but actively seeking employment and currently available to start work

- Even when the economy is growing fast there is a non-zero unemployment rate
 - Frictional unemployment (e.g. searching)
 - Structural unemployment (e.g. obsolete industries)
- The natural rate of unemployment is the rate we would observe in the long run. We will call this state as **Full employment (FE)**.
- Cyclical unemployment is the difference between actual and natural unemployment.
- In this course, we will focus on cyclical unemployment.

- In our model, the labor market clearing implies there is no unemployed.
- Given an equilibrium wage, the part of the supply to the right of the equilibrium, would not be considered part of the labor force (i.e., not looking for a job).
- This implies that our model is not able to replicate any non-zero unemployment rate.
- Some assumptions must be lifted to create a rigidity in the labor market
 - Price rigidity.
 - Imperfect competition.
 - Search and matching.

- We will talk more about this debate in future classes (This is important)
- For now we just need to be clear that when the labor market is in equilibrium:
 - The economy is in *full employment*
 - The unemployment rate is the natural
 - Output is determined by the production function and the equilibrium labor input.
- When the labor market is not in equilibrium, output and employment will depend on the assumptions we make to account for the disequilibrium.

Figure 18: Full employment line

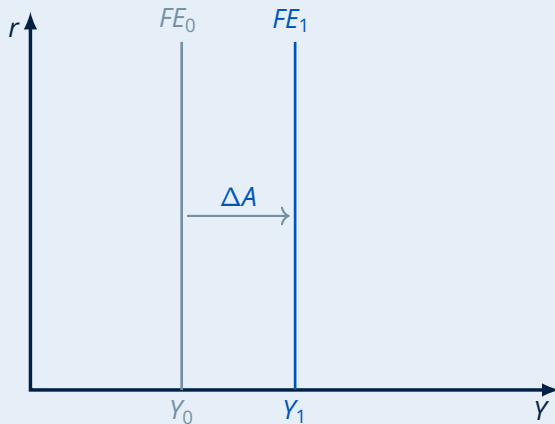


A positive supply shock

- Assume the economy is shocked with a temporary TFP increase.
- This does not affect the labor supply.
- The shock increase the labor demand (the labor demand shift to the right)
- This will imply an equilibrium with higher wages and labor input
- The full employment line shift to the right
- If wages do not adjust quickly, there will be excess of labor demand

A positive supply shock (cont.)

Figure 19: Shift in FE due to TFP shock



Summary

- Firms hire labor in order to maximize profits
- If the labor and goods market are competitive, the firm's labor demand will equalize real wages to marginal productivity (MPN)
- Changes in wages will constitute a movement along the demand curve
- Increases in capital or total factor productivity will shift the labor demand to the right

- Households will offer their time to the firms in exchange for the market real wage. They do so in order to maximize utility.
- The budget and time constraints links consumption, labor, and leisure: the more you work the more you can consume, but the less time you have for leisure.
- The labor supply might backward bend: the income effect could overcome the substitution effect
- We will assume the substitution effect dominates in the short run.
- Although our model is static, we introduced some forward looking elements by interpreting the exogenous income B as the present value of lifetime resources.

- The government influences the decision of the household through taxes and transfers (we assumed government consumption did not have utility implications for the household)
- Proportional taxes distort the optimal consumption/leisure allocation: distortion effect.
- Lump-sum transfers affect only the budget constraint.
- If changes in taxes/transfers are permanent, or they are expected in the future, they might affect the PVLR, shifting the labor supply today.

- Unemployment is not a feature that would arise naturally in our model: we will need extra assumptions.
- Natural unemployment is the one that would prevail in the long run. (Structural + frictional).
- When the economy's unemployment is the natural we are in *full-employment*.
- Unemployment also has a cyclical component. The reasons behind it are part of the debate (price vs quantity rigidities)

References

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